1) Let $A$ be a closed linear operator on $X$ such that $(A - \lambda)^{-1}$ is compact for some $\lambda \in \rho(A)$. Show that $\sigma_c(A)$ is empty.

2) Let $A$ be a closed operator on $X$ such that $0 \in \rho(A)$. Show that if $\lambda \neq 0$ then $\lambda \in \sigma(A)$ iff $1/\lambda \in \sigma(A^{-1})$.

3) Let $A$ be defined on $l^2$ by

$$A(x_1, x_2, ...) = (x_1, 2x_2, ..., nx_n)$$

where $D(A)$ consists of those $x \in l^2$ such that $\sum_{n=1}^{\infty} |nx_n|^2 < \infty$.

a) Is $A$ closed? Is it densely defined? Does $A'$ exist?

b) What are $\sigma(A)$, $\Phi_A$ and $\sigma_c(A)$?

4) Suppose $a \in C(\mathbb{R})$ and let $T_a$ be an unbounded operator on $L^2(\mathbb{R})$ with domain $D(T_a) = \{ f \in L^2 \mid af \in L^2 \}$ and $T_a f = af$. What is the spectrum of $T_a$?

Assume $b \in L^1(\mathbb{R})$ and $C_b(f) = b * f$ (the convolution of $b$ and $f$). Prove that $C_b$ is a bounded operator on $L^2(\mathbb{R})$. What is its spectrum.

5) Consider the operator $T = \frac{d}{dx}$ on $C[a, b]$. We see it as an unbounded operator with domain $D(T) = C^1[a, b]$.

Also consider

$T_1 = \frac{d}{dx}$ with $D(T_1) = D(T) \cap \{ u \mid u(a) = 0 \}$

$T_2 = \frac{d}{dx}$ with $D(T_2) = D(T) \cap \{ u \mid u(b) = 0 \}$

$T_3 = \frac{d}{dx}$ with $D(T_3) = D(T) \cap \{ u \mid u(a) = u(b) = 0 \}$

Find the spectrum of the operators $T, T_1, T_2$ and $T_3$. 