1) Show that a linear functional $A$ on a B-space is bounded if and only if $N(A)$ is closed.

2) Let $T \in B(X,Y)$. Then $T$ is compact if and only if $[T] \in B(X/N(T), Y)$ is compact.

3) Let $X = l^2$. Define $T \in B(X)$ by $T(x_1, x_2, ...) = (0, x_1/1, x_2/2, ..., x_n/n, ...)$. Prove that $T$ is compact. Extra question: Prove that it has no eigenvalues.

4) Let $k(x, y)$ be a continuous function on $[0,1]^2$. Define $Tf$ by

$$Tf(x) = \int_0^1 k(x, y)f(y)dy$$

for any integrable function on $[0,1]$. Show that for any $1 \leq p \leq \infty$, $T$ is compact from $L^p(0,1)$ to itself.

5) Let $N$ be a finite dimensional subspace of a normed vector space $X$. Prove that there exists a closed subspace $X_0$ such that $X = X_0 \oplus N$.

6) Let $R$ be a closed subspace of a normed vector space $X$ such that $R^\circ$ is of finite dimension $n$. Prove that there is an $n$ dimensional subspace $N$ of $X$ such that $X = R \oplus N$. \