Meridional Momentum Flux and Superrotation in the Multiscale IPESD MJO Model

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ABSTRACT

The derivation of the meridional momentum flux arising from a multiscale horizontal velocity field in the intraseasonal, planetary, equatorial synoptic-scale dynamics (IPESD) multiscale models of the equatorial troposphere is presented. It is shown that, because of the balance dynamics on the synoptic scales, the synoptic-scale component of the meridional momentum flux convergence must always vanish at the equator. Plausible Madden–Julian oscillation (MJO) models are presented along with their planetary-scale meridional momentum fluxes. These models are driven by synoptic-scale heating fluctuations that have vertical and meridional tilts. Irrespective of the sign of the synoptic-scale meridional momentum flux (direction of the tilts) in each of the four MJO examples, the zonal and vertical mean meridional momentum flux convergence from the planetary scales always drives westerly winds near the equator: this is the superrotation characteristic of actual MJOs. The concluding discussion demonstrates that equatorial superrotation occurs when the planetary flow due to the vertical upscale momentum flux from synoptic scales reinforces the horizontally convergent flow due to planetary-scale mean heating.

1. Introduction

The dominant component of intraseasonal variability in the Tropics is the 40–50-day tropical intraseasonal oscillation, often called the Madden–Julian oscillation (MJO) after its discoverers (Madden and Julian 1972). In the troposphere, the MJO is an equatorial planetary-scale wave envelope of complex multiscale convective processes that propagates across the Indian Ocean and western Pacific at a speed of roughly 5 m s\(^{-1}\) (Nakazawa 1988; Hendon and Salby 1994; Hendon and Leimbann 1994; Maloney and Hartmann 1998). The planetary-scale circulation anomalies associated with the MJO significantly affect monsoon development, intraseasonal predictability in midlatitudes, and impact

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It is widely recognized that organized convection on multiple scales is an essential ingredient of the Madden–Julian oscillation (Inness et al. 2001; Lin and Johnson 1996; Moncrieff and Klinker 1997; Grabowski and Moncrieff 2001; Slingo et al. 2003; Sperber 2003; Wang 2005; Zhang 2005). This organization is manifested as correlations between different components of the dynamical fields (i.e., velocity or temperature), which yield net upscale fluxes in those fields and drive the planetary-scale organized flow. Of particular interest is the tilted convection, which occurs from the submesoscale up to the planetary scale in the Tropics. Depending on the scale in question (i.e., squall lines or organized superclusters), these tilts are either predominantly meridional/zonal (Moncrieff 2004) or vertical/zonal (Lin and Johnson 1996; Moncrieff and Klinker 1997; Biello and Majda 2005). Through Reynolds stresses, these tilts can generate an average zonal momentum flux convergence on the planetary scales.
In a series of papers that used the systematic multiscale intraseasonal, planetary, equatorial synoptic-scale dynamics (IPESD) theory of Majda and Klein (2003), Majda and Biello have constructed a family of models describing the spatial structure of the Madden–Julian oscillation, which contain several features in common with the observational record. The papers, which introduced the multiscale models for the MJO (Majda and Biello 2004, hereafter MB04), developed a realistic MJO theory (Biello and Majda 2005, hereafter BM05), and developed variations and generalizations of that theory (Biello and Majda 2006, hereafter BM06). Unlike the linear Matsuno–Gill models (Matsuno 1966; Gill 1980), the IPESD models explicitly incorporate the synoptic-scale momentum and temperature flux convergence into the equations for the planetary scale flow. These models utilize aspects of the observed latent heating on the synoptic scale to drive balanced flows on this scale while their upscale fluxes and planetary-scale mean heating drive the planetary-scale flow associated with the Madden–Julian oscillation.

In particular, the canonical MJO models in BM05 use the fact that superclusters are observed to be vertically/zonally tilted westward with height (Nakazawa 1988; Straub and Kiladis 2003; Wheeler and Kiladis 1999; Wheeler et al. 2000). The planetary-scale forcing in the canonical model is due to the planetary-scale mean of the latent heat release, in addition to the flux convergence; primarily the vertical component of the zonal momentum flux convergence, \(\overline{uw}\). BM06 considers other variants of the IPESD–MJO models, which contain both purely meridionally tilted MJO models and different combinations of vertical and meridional tilt. Nonetheless, the canonical, vertically tilted model is able to capture much of the planetary-scale flow structure of observed MJOs in the simplest formulation (i.e., fewest number of parameters).

This paper has three objectives. The first is to understand nonlinear upscale fluxes in the context of a multiscale model, such as IPESD. In particular, the separation of scales in IPESD means that there is an explicit upscale flux from the synoptic scales, but there is also an upscale flux from the planetary scale that affects the mean climatology. We show that these contributions can be superposed and that the asymptotic model provides a formal ordering for the relative sizes of each contribution. The second objective is to understand the vertically and zonally averaged meridional momentum flux convergence in IPESD MJO models that differ in the nature of their meridional and vertical tilts. The four models describe purely vertically tilted flows, purely meridionally tilted flows, and two hybrids that contain both vertical tilts and eastward and westward meridional tilts, respectively.

The vertically/zonally averaged meridional momentum flux convergence is the only term in the dynamical equations that drives the zonal mean barotropic flow, and therefore, drives the climatological mean winds. The final objective is to understand the equatorial superrotation, which is a direct consequence of the mean meridional momentum flux at the equator. Superrotation is observed during an MJO event (Zhang 2005; Moncrieff 2004) where the vertically/zonally averaged equatorial winds are westerly. We shall show that there is no equatorial superrotation due to the synoptic-scale meridional momentum flux convergence in the IPESD models. Therefore, superrotation is a consequence of organization on the planetary scales alone, and its presence is quite insensitive to the structure of the synoptic-scale meridional momentum flux.

The paper is organized as follows. Section 2 contains a brief overview of the IPESD theory of the MJO along with a derivation of the expressions for the momentum flux arising from the synoptic and planetary scales. It concludes with explicit expressions for the upscale fluxes of temperature and momentum, which arise from latent heat release on the synoptic scales. In section 3 we consider four plausible MJO examples, each with a different (or no) synoptic-scale meridional momentum flux convergence. The concluding discussion in section 4 shows how equatorial superrotation is a consequence of correlated planetary-scale heating plus the vertical component of the upscale flux of zonal momentum from the synoptic scales.

2. The IPESD theory of the MJO

The IPESD theory (Majda and Klein 2003) is a systematic multiscale theory for the equatorial troposphere that describes forced synoptic- and planetary-scale flows and their coupling through upscale fluxes of momentum and temperature. The scales of each of the terms in the IPESD models are summarized in Table 1. The total flow consists of planetary-scale mean plus synoptic-scale fluctuations

\[
\begin{align*}
\theta &= \epsilon \theta'(\epsilon x, y, z, t) + \epsilon^3 \theta(\epsilon x, y, z, t) + O(\epsilon^5) \\
p &= \epsilon p'(\epsilon x, y, z, t) + \epsilon^3 p(\epsilon x, y, z, t) + O(\epsilon^5) \\
u &= u'(\epsilon x, y, z, t) + \epsilon^3 \bar{u}(\epsilon x, y, z, t) + O(\epsilon^5) \\
v &= v'(\epsilon x, y, z, t) + \epsilon^3 \bar{v}(\epsilon x, y, z, t) \\
w &= w'(\epsilon x, y, z, t) + \epsilon^3 \bar{w}(\epsilon x, y, z, t) + O(\epsilon^5)
\end{align*}
\]

which vary on synoptic scales \(x\) and \(y\), in the vertical, \(z\) and on intraseasonal time scales, \(t\). The single asympt-
The scales and nondimensional parameters of the IPESD model. Square parentheses indicate that the value of one unit of the nondimensional variable corresponds to given scale.

<table>
<thead>
<tr>
<th>Physical quantity</th>
<th>Name</th>
<th>Value or unit scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Froude number</td>
<td>( \epsilon )</td>
<td>0.1</td>
</tr>
<tr>
<td>Gravity wave speed</td>
<td>( c )</td>
<td>50 m s(^{-1})</td>
</tr>
<tr>
<td>Equatorial time scale</td>
<td>( T_E )</td>
<td>((c \beta)^{-1/2} = 8.3) h</td>
</tr>
<tr>
<td>Equatorial deformation</td>
<td>( l_\epsilon )</td>
<td>((c \beta)^{1/2} = 1500) km radius</td>
</tr>
<tr>
<td>Troposphere height</td>
<td>( H_p )</td>
<td>16 km</td>
</tr>
<tr>
<td>Synoptic-scale dimensions</td>
<td>([x, y])</td>
<td>(l_s = 1500) km</td>
</tr>
<tr>
<td>Vertical dimension</td>
<td>([z])</td>
<td>(H_f/\pi \approx 5) km</td>
</tr>
<tr>
<td>Zonal planetary scale</td>
<td>([X])</td>
<td>(l_z/\epsilon \approx 1500) km</td>
</tr>
<tr>
<td>Planetary advection time</td>
<td>([t])</td>
<td>(T_p/\epsilon \approx 3) days</td>
</tr>
<tr>
<td>Horizontal velocity scale</td>
<td>([u', v', \vec{U}])</td>
<td>(\epsilon c = 5) m s(^{-1})</td>
</tr>
<tr>
<td>Vertical velocity scale</td>
<td>([w'])</td>
<td>(\epsilon H/\epsilon = 2.5) cm s(^{-1})</td>
</tr>
<tr>
<td>Temperature scale</td>
<td>([\theta^\prime, \vec{\theta}])</td>
<td>3 K</td>
</tr>
<tr>
<td>Pressure scale</td>
<td>([p^\prime, \vec{P}])</td>
<td>(\epsilon \sigma^2 = 250) (m s(^{-1}))^2</td>
</tr>
<tr>
<td>Synoptic-scale heating rate</td>
<td>([S^\prime])</td>
<td>10 K day(^{-1})</td>
</tr>
<tr>
<td>Planetary-scale heating rate</td>
<td>([\vec{S}])</td>
<td>1 K day(^{-1})</td>
</tr>
<tr>
<td>Momentum drag rate</td>
<td>(d_0)</td>
<td>(T_p/5) (days)(^{-1}) \approx 0.55</td>
</tr>
<tr>
<td>Thermal dissipation rate</td>
<td>(d_\phi)</td>
<td>(T_p/15) (days)(^{-1}) \approx 0.18</td>
</tr>
</tbody>
</table>

The Froude number \( \epsilon \) arises by taking a distinguished limit in the fully nonlinear forced equatorial primitive equations. It is both a measure of the small ratio of synoptic-scale heating fluctuations to the planetary mean heating and of the Froude number of the synoptic-scale fluctuations. Furthermore, it is also a measure of the separation of scales between zonal synoptic scales, \( x \) and the zonal planetary scale

\[
X = \epsilon x. \tag{2.2}
\]

The variables are scaled so that \(-10/3 \leq y \leq 10/3\), \(0 \leq z \leq \pi\), and \(-4/3 \leq X \leq 4/3\) describes the whole zonal extent of the equatorial troposphere from \( \pm 5000\) km meridionally and 16 km vertically. The unit of time is 3.3 days, which is useful in describing intraseasonal variations and \( \epsilon \approx 0.1\) means that the unit of synoptic variation is one-tenth of the unit of zonal planetary-scale variation. Faster variations have been disregarded but can be included without significant differences as in BM06. The velocity is measured in units of 5 m s\(^{-1}\) in the horizontal and 0.016 m s\(^{-1}\) in the vertical direction and the potential temperature is measured in units of 3.3 K.

Though the IPESD models allow for general thermal and momentum forcing, the MJO models in MB04, BM05, and BM06 only consider direct forcing arising through the release of latent heat of condensation. Momentum forcing of the planetary-scale flows arises naturally due to the convergence of upscale fluxes from the synoptic scales: the zonal mean of the momentum flux convergence arising from synoptic-scale flows. Tropospheric Newtonian cooling and momentum drag are also included in the models and because of the several day dissipation times associated with them (Bretherton and Sobel 2003; Lin et al. 2005), affect the planetary-scale, not the synoptic-scale dynamics. The heating anomaly is separated into its zonal synoptic-scale fluctuating component and a weaker zonal planetary-scale mean

\[
S_\phi = S^\prime_p(X, x, y, z, t) + \epsilon \vec{S}_p(X, y, z, t), \tag{2.3}
\]

and is measured in units of 10 K day\(^{-1}\).

The equations of the IPESD theory are as follows. The synoptic scales are described by balanced dynamics that are forced by the synoptic-scale heating fluctuations

\[
-\epsilon y u' + p_z' = 0
\]

\[
y u' + p_x' = 0
\]

\[
w' = S_\phi, \quad \vec{S}_p = 0
\]

\[
p_z' = 0
\]

\[
u_z' + v_z' + w_z' = 0 \tag{2.4}
\]

with the rigid lid top and bottom boundary conditions \( w' = 0 \) at \( z = 0, \pi \). The planetary scales are driven by the upscale momentum and thermal flux convergences due to the synoptic-scale dynamics, and by the mean heating on planetary scales

\[
\vec{U}_x - y \vec{V} + \vec{P}_X = F^U - d_0 \vec{U}
\]

\[
y \vec{U} + \vec{P}_y = 0
\]

\[
\vec{S}_x + \vec{W} = F^\theta - d_0 \vec{S} + \vec{S}_\phi
\]

\[
\vec{P}_z = \vec{\Theta}
\]

\[
\vec{U}_x + \vec{V} + \vec{W}_z = 0, \tag{2.5}
\]

where the flux convergences are

\[
F^U = -\left(\epsilon \sigma'\right)_x - \left(\epsilon \sigma'\right)_z
\]

\[
F^\theta = -\left(\epsilon \sigma'\right)_y - \left(\epsilon \sigma'\right)_z, \tag{2.6}
\]

and, again, rigid lid boundary conditions are used at the top and bottom of the troposphere \( \vec{W} = 0 \) at \( z = 0, \pi \). The IPESD models with a general tropospheric stratification and lower boundary layer dissipation were considered in BM06, but for the purposes of this paper, it suffices to consider the simpler, Boussinesq IPESD model outlined above.

\[a. \text{Definition of momentum flux in the multiscale theory}\]

We are interested in the zonal mean of the zonal momentum flux vector due to the total flow from all scales in our multiscale theory,
where the angle brackets signify the zonal mean and \( C_E \) is the circumference of the earth at the equator (actually the circumference measured on an equatorial \( \beta \) plane). The ansatz of the IPESD model is that the velocities can be separated into their zonal planetary-scale means plus synoptic-scale fluctuations. In the model, the zonal velocity has a mean that is comparable to its fluctuations whereas the planetary-scale means of the vertical and meridional velocities are small compared to their synoptic-scale fluctuations. The separation of scales is given in Eq. (2.1) where the synoptic-scale means are defined as

\[
\bar{g}(X, y, z, t) = \lim_{L \to \infty} \frac{1}{2L} \int_{-L}^{L} g(x, X, y, z, t) \, dx,
\]

and a consequence of the separation of the flow into mean and fluctuation is that the zonal mean of the fluctuation vanishes for each field,

\[
\bar{u'} = \bar{v'} = \bar{w'} = 0.
\]

In a multiscale asymptotic theory, the assumption of two separated length scales is tantamount to adding one more dimension over which functions can vary. Basically at every point of the planetary-scale coordinate, \( X \), there is a whole space of the synoptic-scale coordinate, \( x \in [-L, L] \). The value of a function, \( g(x, X) \) at a point \( X \) is simply the mean over the synoptic-scale \( x \); that is, \( g(x, \bar{X}) \) and, using this intuition, we can understand how to compute the true zonal mean of a function which varies on the two length scales, \( x, X \). Therefore, the zonal synoptic-scale average of a function for which a multiscale ansatz has been made is expressed as a double integral over its counterpart that has been assumed to vary over two scales

\[
\frac{1}{C_E} \int_{0}^{C_E} g(x) \, dx \to \frac{1}{C_*} \int_{0}^{C_*} g(x, X) \, dX
\]

\[
= \frac{1}{C_*} \frac{1}{2L} \int_{0}^{C_*} \int_{-L}^{L} g(x, X) \, dx \, dX.
\]

where \( C_* \) is the earth’s circumference expressed in units of the planetary scale, \( C_* = \sqrt{C_E / L_E} \approx 3.33 \). The limit of \( L \) going to infinity has been dropped in the last expression because it suffices to consider synoptic-scale structures that are periodic in \( L \); and for practical computations, this is what is done. We can now calculate the zonal mean momentum flux from the IPESD multiscale theory

\[
\langle F^U \rangle = -\frac{1}{C_*} \int_{0}^{C_*} \left[ \langle u' \rangle \langle v' \rangle + (\epsilon \bar{U}) \right] \, dX
\]

\[
= -\frac{1}{C_*} \int_{0}^{C_*} \left[ \langle u' \rangle \langle v' \rangle + \epsilon \bar{U} \right] \, dX
\]

\[
= -\frac{1}{C_*} \int_{0}^{C_*} \left[ \langle u' \rangle \langle v' \rangle + \epsilon \bar{U} \right] \, dX
\]

where the vanishing of the planetary-scale mean of the synoptic-scale fluctuations has been used to simplify the last expression. Notice that there are two contributions to the meridional component of the flux and two contributions to the vertical component, one from each of the planetary and synoptic scales. Furthermore, each variable in Eq. (2.11) is order one in nondimensional units: except \( \epsilon \), which gives us an asymptotic ordering. Because of the IPESD scaling of the planetary mean vertical and meridional flow, both vertical and meridional fluxes are dominated (at least formally) by their synoptic-scale fluctuations: the planetary mean contribution is weaker by a factor of \( \epsilon \).

In the MJO models of MB04, BM05, and BM06, the synoptic-scale fluctuations themselves are concentrated in a moving envelope of convective activity that only covers 10 000 km longitudinally, whereas the planetary-scale response is global. Therefore, after taking the zonal average, it remains to be seen whether or not the zonal mean flux owing to the synoptic-scale fluctuations actually dominates the component due to the planetary-scale flow. In fact, as we shall see, the relative contribution of these two components depends on the details of the synoptic-scale flow in a variety of examples, all of which give a plausible structure for the MJO.

b. MJO models

Following BM05 and BM06, we consider latent heat release forcing the first two vertical baroclinic modes in the IPESD models since these two modes can capture the large-scale effects of congestus, deep, and stratiform clouds (Majda et al. 2004; Lin and Johnson 1996). In BM05 and BM06, it was shown that, since the synoptic-scale-balanced dynamics are linear in the latent heat fluctuations then it suffices to consider a superposition of zonal modes in the synoptic-scale structures. Furthermore, since the upscale fluxes are zonal averages of the synoptic-scale flows it suffices to consider only one zonal synoptic-scale wavenumber; the nonlin-
ear upscale transport arising from different synoptic-scale zonal wavenumbers averages to zero. Therefore a general heating profile consisting of a single synoptic-scale mode, $k$, can be expressed as

$$S_0(x, y, z, X, t) = H_1(y) \cos[kx + \phi_1(y)]\sin(z)$$
$$+ H_2(y) \cos[kx + \phi_2(y)]\sin(2z)$$

(2.12)

and the corresponding planetary-scale mean heating is

$$\mathcal{S}_0(y, z, X, t) = \mathcal{H}_1(y) \sin(z) + \mathcal{H}_2(y) \sin(2z),$$

(2.13)

where all of the functions also depend on the planetary-scale zonal coordinate, $X$, and time, $t$. Typically, the meridional structure is equatorially confined and was modeled in MB04, BM05, and BM06 using Gaussians with widths on the order of a few thousand kilometers. BM05 discussed the effect of the various terms in these functions, in particular focusing on a constant (in $y$) phase lag of the second baroclinic mode with respect to the first. This was shown to beget a vertical/zonal tilt in the synoptic-scale heating profile, which can model both westward tilted superclusters and lower troposphere congestus heating when the phase lag is constant in $y$.

When the phase lag is nonconstant in $y$, then each of the vertical components of the baroclinic heating has, itself, a meridional tilt along the curve $kx + \phi(y) = constant$. The effect on the MJO models, considered in MB04 and BM05, of these meridional tilts is the introduction of a meridional momentum flux from the synoptic scales, $(u'w')$, and was considered in BM06.

In this paper, we are interested in the variation of the total meridional transport of zonal momentum in the MJO models that, as was shown in Eq. (2.11), splits into a synoptic-scale and planetary-scale component. The IPESD models have the advantage that the synoptic-scale balanced dynamics in Eq. (2.4) can be explicitly solved in terms of the synoptic-scale heating fluctuations. Consequently, the upscale momentum and temperature flux can also be expressed analytically in terms of the heating fluctuations (2.12). The planetary-scale components of the fluxes, which are formally weaker than the synoptic scales, can only be diagnosed after numerically solving Eq. (2.5) for a specific heating profile.

Using $\Delta = \phi_2(y) - \phi_1(y)$ to express the phase lag and subscript $y$ to denote differentiation, we arrive at analytic expressions for the meridional component of the zonal momentum flux,

$$\overline{(w'u')} = -\frac{y^2}{4k} \left\{ yH_1^2 \phi_{1y} \sin(2z) + 2yH_2^2 \phi_{2y} \sin(4z) + \left[ yH_1H_2 \sin(\Delta) (\phi_{1y} - 2\phi_{2y}) + yH_1H_2 \cos(\Delta) (\phi_{1y} + 2\phi_{2y}) \right] \sin(z) \right\}. \quad (2.15)$$

and the vertical component of the zonal momentum flux,

$$\overline{(w'\theta')} = -\frac{3y^2}{4k} H_1H_2 \sin(\Delta) [\cos(z) - \cos(3z)]. \quad (2.16)$$

It was also shown in BM05 and BM06 that, since the synoptic-scale balanced dynamics yield small temperature perturbations near the equator, the temperature flux is much less relevant than the momentum flux in the equatorially symmetric MJO models. Though we shall not explicitly concentrate on their effects, for completeness we nonetheless record the meridional and vertical components of the temperature flux

$$\overline{(w'\theta')} = -\frac{y^3}{2k} H_1H_2 \sin(\Delta) [\sin(z) - \sin(3z)], \quad (2.16)$$

$$\overline{(w'\theta')} = -\frac{3y^2}{4k} H_1H_2 \sin(\Delta) [\cos(z) - \cos(3z)]. \quad (2.17)$$

If there is a lag between the first and second baroclinic modes, $\Delta \neq 0$, the flow is zonally/vertically tilted and drives both a vertical momentum flux and a temperature flux; though the latter is weaker because of its cubic dependence on $y$ near the equator. In order for a nonzero meridional momentum flux, either the meridional extent of the two baroclinic modes must differ, $H_1H_2 - H_2'H_2 \neq 0$, or the phase lines of the either baroclinic mode must be meridionally tilted, $\phi_{1y} \neq 0$.

Notice that of all of these effects, the only one that drives a vertically averaged forcing is the meridional convergence of the barotropic component of the zonal momentum flux, the term independent of $z$ in Eq. (2.14). That is to say, in a vertically bounded troposphere, vertically and zonally averaged winds can only be driven by the meridional flux of momentum from higher latitudes; vertical momentum flux can redistribute the wind shear, but not drive the mean winds. If the
meridional derivative of $-\overline{(u'v')}$ is positive (negative) then there is a mean westerly (easterly) wind forcing. Multiplying the velocity correlation $\overline{(u'v')}$ by a typical tropospheric density, which for the Boussinesq models is simply the density of air at the base of the troposphere, yields a force per unit area driving the zonal flow which can be compared quantitatively with numerical results on the MJO (Grabowski 2003).

The driving of a planetary zonally and vertically averaged westerly flow near the equator is a feature of observed MJOs called superrotation and the scale and structures from which it arises remain unresolved issues (Moncrieff 2004). However, it was shown in that work that meridional momentum flux convergence in the superrotation context requires eddies to be asymmetric in the zonal direction; whether using open streamlines as in (Moncrieff 2004) or meridional tilts as discussed herein. The framework outlined above details a clear way to separate the scale of the barotropic driving and, thus, determine which specific structures within the flow (such as the synoptic-scale superclusters, or the planetary-scale trailing Rossby gyres) drive the mean westerly wind.

3. The meridional momentum flux in models of the MJO

In this section we consider four examples of heating profiles that drive plausible MJO-like flows on the planetary scales. The first two “canonical” examples were considered elsewhere in BM05 and BM06, whereas the second two examples are perturbations of these canonical examples.

The MJO is an envelope of convective activity moving eastward at about 5 m s$^{-1}$ from the Indian Ocean through the western Pacific and centered about the equator. Furthermore, the passing of an MJO event is characterized first by lower troposphere congestus convection, followed by deep convection and then stratiform clouds (Johnson and Lin 1997; Kiladis et al. 2005; Lin and Johnson 1996; Houze et al. 2000; Yanai et al. 2000). Using this qualitative structure, the MJO models consist of an envelope of latent heating, $S^p$, moving eastward at an imposed rate of 5 m s$^{-1}$ driving the synoptic- and planetary-scale flows to equilibrium. For all of the examples, the planetary mean heating is chosen to correspond to the congestus/deep/stratiform paradigm by allowing the heating maximum to be in the lower half of the troposphere in the eastern portion of the moving envelope, in the middle of the troposphere at the center of the envelope and in the upper half of the troposphere in the western portion. The mean heating is given by a stationary envelope, and in the moving reference frame is given by

$$S_b(y, z, X) = F(X)e^{-y^2/2}[\sin(z) - \alpha(X) \sin(2z)],$$

(3.1)

where the envelope is the first positive portion of a cosine of 10 000-km zonal extent and $\alpha$ is an order one linear function in $X$, which interpolates the mean heating from the congestus to the stratiform portion of the envelope.

Though all of the examples use the same planetary-scale mean heating and the same strengths of synoptic-scale heating fluctuations, they differ in the shape of those fluctuations and therefore in the synoptic-scale flows and upscale fluxes, which they drive. The synoptic-scale fluctuating heating is given by

$$S'_h(x, y, z, X, t) = F(X) e^{-y^2/2}[\cos(x + y|y|) \sin(z) - \alpha(X) \cos(x + y|y| + \phi) \sin(2z)],$$

(3.2)

where the envelope function, $F$, is the same as in the mean profile. Again, $\alpha(X)$ expresses whether or not the heating fluctuations are centered in the upper half of the troposphere ($\alpha > 0$) or lower half of the troposphere ($\alpha < 0$). A constant positive (negative) phase lag of the second baroclinic mode with respect to the first, $\phi$, expresses an upward/westward (eastward) tilt for a heating maximum in the upper troposphere; as in BM05, this models westward tilted superclusters with deep convective and stratiform heating. Upward/westward tilted structures drive lower troposphere westerlies and, therefore, upper troposphere easterlies.

Synoptic-scale meridional tilts of either the first or second baroclinic modes are expressed by $\phi(y) = \gamma|y|$ and we restrict attention to the cases that both baroclinic modes have the same meridional tilt. The phase curves are straight lines tilted westward (eastward) from the equator in both hemispheres for $\gamma > 0$ (< 0) and driving westerly (easterly) mean winds at the equator through meridional convergence (divergence) of westerly momentum.

As an important diagnostic, we will use the vertical average of the zonal/meridional velocity correlation from both the synoptic- and planetary-scale flows. The synoptic-scale contribution to this quantity is simply the barotropic component from Eq. (2.14) and using the particular heating in Eq. (3.2) it is expressed as

$$\overline{(u'v')} = -\frac{\gamma y^2 H^2 \text{sgn}(y)}{4k},$$

(3.3)
where \( \text{sgn}(\gamma) \) is the signum function. It is clear that in these models, as in any models that use the IPESD framework, the upscale flux of meridional momentum is, at best, proportional to the square of latitude near the equator. Therefore, at the equator, there is no driving of mean zonal winds due to upscale flux convergence from latent heating on the synoptic scales. This fact is a direct consequence of the weak temperature gradient approximation and balanced dynamics on the synoptic scale from Eq. (2.4). The IPESD models can accommodate situations where the upscale meridional momentum flux does not vanish at the equator, but then an explicit momentum forcing on the synoptic scales would have to be included; there is no evidence for such a forcing.

Therefore, equatorial superrotation, if it is to arise in the IPESD MJO models, must occur because of planetary-scale organization alone; the synoptic scales can drive mean zonal winds, but they will always be zero at the equator. The contribution of the synoptic-scale fluxes to the mean wind away from the equator can be significant, however, as can be seen from the expression (3.3) and the fact that the planetary-scale contribution to the fluxes is higher order, as is evident from Eq. (2.11).

Summarily, all of the four examples use the same mean heating profile and have the following synoptic-scale features:

1) The first example is the canonical, vertically tilted MJO model introduced by BM05. The synoptic-scale flow contains no meridional tilt, \( \gamma = 0 \), has a synoptic-scale profile with westward, vertically tilted superclusters in the western portion of the convective envelope and lower troposphere congestus in the eastern portion of the envelope. In this example, the meridional momentum flux vanishes and the value of the other flux convergences in the congestus portion of the envelope is the negates of their values in the supercluster portion of the envelope.

2) In the second example, only the western supercluster portion of the envelope contributes to the upscale flux, and then only through meridional tilts in the synoptic scale heating profile. A very similar example (albeit meridionally broader) was introduced in BM06 and was inspired by the meridionally tilted model used by (Moncrieff 2004) to explain convective organization in numerical models of the MJO (Grabowski 2001). We use only the first baroclinic mode to describe the heating and the parameters, \( \gamma = 1/2 \) and \( k = 1 \), which correspond to a 2:1 meridional:zonal tilt in the synoptic-scale heating contours. Though there remains some amount of vertical momentum flux, this example is dominated by the meridional transport of zonal momentum. Furthermore, the upscale temperature flux is identically zero.

3) The third example is a hybrid of the first two cases containing both vertically and meridionally tilted superclusters in the western half of the envelope. Whereas, the flow in the eastern portion of the envelope has no meridional tilt, the same form as in example 1.

4) The final example again uses vertically tilted structures from example 1 and meridional tilts. However, in this case, the meridional tilts are eastward, \( \gamma = -1/2 \), and occur throughout the whole convective envelope.

a. Canonical MJO model with vertical tilts alone

The horizontal flow at four heights in the troposphere for the canonical MJO model of BM05 (example 1) is shown in Fig. 1 and contours of the zonal velocity above the equator are shown in Fig. 2. As discussed in BM05, these have several features in common with the observational record of the MJO.

Figure 3 shows the meridional flux of zonal momentum \( \langle \overline{u} \rangle \) for the total flow (solid), the planetary-scale contribution (dot/dash) and the synoptic-scale contribution (dash). The fluxes have been reexpressed in units of force per unit area to provide an easier route for comparison with numerical simulations and to emphasize that, upon reconstituting the order one fields which contribute to the total flux, the planetary-scale contribution is \( O(\epsilon) \) smaller than the synoptic scales.

It is immediately evident from Fig. 3 that the canonical MJO model admits superrotation at the equator even though there is no upscale meridional momentum flux due to meridionally tilted flows on the synoptic scales. Since \( \langle \overline{u} \rangle_x < 0 \), the troposphere within about 500 km of the equator is rotating faster than the solid earth. This is an especially interesting discovery since the model was not originally designed with superrotation in mind.

Over 80% of the planetary-scale flux is due to the first baroclinic mode, which emphasizes the importance of this mode, even though the second and third baroclinic modes are needed in order to reproduce the correct horizontal flow structure. The amount of the contribution is determined by calculating the correlation of the total flux with each of the vertical modes. In the discussion we shall show how superrotation at the equator requires a correlation of planetary-scale heating and upscale momentum flux in the same baroclinic mode.
Since the temperature forcing on the planetary scales is in the first and second baroclinic modes whereas the upscale momentum flux convergence drives only the first and third modes, it is clear that their correlation must be dominated by the first baroclinic mode.

**b. Westward meridionally tilted superclusters**

Figures 4 and 5 show the flow in the horizontal plane and the zonal flow above the equator for example 2 consisting of westward meridionally tilted superclusters. In this case, the westerly wind burst sets in at the base of the troposphere at the equator. In the western portion of the envelope, the trailing Rossby gyres migrate poleward in the lower portions of the troposphere causing the jet associated with the westerly wind burst to split in two.

Since there is no upscale flux from the congestus heating, the structure in the eastern portion of the envelope is different from that of example 1. First, there is no anticyclone pair leading the westerly wind burst at the base of the troposphere. Second, the westerly outflow in the upper half of the troposphere only has the meridionally broad Kelvin component in the east, as opposed to the narrowly confined jet evident in the first example; compare Figs. 1d and 4d.

On the other hand, the westerly wind burst in this example has a structure that is reasonably similar to the first example and, consequently, to observed MJOs.
The vertical structure of the wind burst above the equator (Fig. 5) shows that the wind burst sets in at the base of the troposphere below easterlies and tilts westward with height. Though the maximum of the wind burst is attained at the base of the troposphere, its value is not significantly different than the maximum westerlies at 5 km.

The synoptic-scale meridional tilts generate upscale meridional momentum flux, which makes the total meridional momentum flux look significantly different than the canonical example. Figure 6 shows the meridional momentum flux split into its synoptic- and planetary-scale contributions. The synoptic-scale meridional momentum flux (dashed line) has vanishing gradient near the equator and attains its maximum at about 1000 km poleward. The planetary-scale contribution (dot–dash) is much smaller than the upscale flux and it attains its first maximum much closer to the equator. Notice that the divergence of the planetary-scale meridional momentum flux is nonzero and westerly in this example, but it is also much smaller than in example 1 near the equator.

Therefore, not only does the synoptic-scale meridional flux convergence vanish at the equator for any IPESD model, the purely meridionally tilted MJO model is also unable to organize planetary-scale flows with a significant meridional flux convergence at the equator. Though there is superrotation in the Tropics in this case, the superrotation right at the equator is much weaker than in the canonical MJO.

A clue to this apparently paradoxical behavior comes from the fact that the second baroclinic mode contributes most to the planetary-scale flux (67% correlation). The meridionally tilted synoptic-scale flux only drives the first baroclinic mode in this example. Therefore, the upscale flux convergence only drives the barotropic and second baroclinic mode, and only in the western portion of the envelope. As discussed above, the planetary-scale mean heating drives both first and second baroclinic modes.

Therefore, the meridional momentum flux is the correlation of meridional and zonal flows arising upscale forcing of the barotropic and second baroclinic mode along with direct forcing of the first and second baroclinic modes. The vertical average of these correlations only retains the correlation of each of the barotropic, first and second baroclinic modes with themselves. We shall show below that planetary-scale equatorial superrotation requires a correlation between the direct heating (of a particular baroclinic mode) with the upscale momentum flux (of the same baroclinic mode). The weakness of the equatorial superrotation in this example, as compared to the canonical MJO, is due to the fact that there is no upscale flux driving the first baroclinic mode, which is the primary component of the planetary-scale heating. Clearly the contribution from the second baroclinic mode is weaker since it is confined to the western portion of the envelope.

c. Hybrid MJO model combining vertical tilts and westward meridionally tilted superclusters

The flow, which results from combining westward meridional tilts with the canonical MJO model, is
shown in Figs. 7 and 8, while the resulting meridional momentum flux is shown in Fig. 9. Features from both of the previous examples are evident in this example: the vertical/westward tilt in the zonal winds; the trailing, split Rossby gyres; and an equatorially confined outflow in the upper half of the troposphere, which has both an easterly and westerly jet. Again, all of these features compare well to the observational record.

Superposing the two previous models seems to result in essentially superposed flows, and, also, superposed meridional momentum flux as is evident in Fig. 9. The synoptic-scale component is meridionally broad with a maximum near ±1000 km, and has a vanishing convergence at the equator. The planetary-scale component is now stronger than in example 1 and, like example 2, is dominated by the first baroclinic mode and yields a significant equatorial superrotation.

d. Canonical MJO model plus eastward meridional tilts

The previous examples suggest that equatorial superrotation exists irrespective of the synoptic-scale meridional momentum flux and is quite insensitive to that flux. In this example we push this hypothesis to the extreme by taking both the westward tilted superclusters and the congestus heating fluctuations from example 1 and tilting them meridionally eastward.

The results are shown in Figs. 10–12 and, again, the...
velocity field is a plausible MJO. The westerly wind burst is westward/vertically tilted, but now it is much more equatorially confined than in any of the previous examples. There is also some anticyclonic rotation in the eastern portion of the envelope, and a clear cyclonic Rossby gyre in the west. As in the canonical example, there is an equatorially confined outflow in the upper troposphere, which is bidirectional.

We can immediately see from Fig. 12 that, though the meridional momentum flux convergence is now, on average, easterly within 1000 km of the equator, immediately around the equator the flux convergence is westerly. In this case, the upscale flux (dashed curve) is easterly because of the eastward meridional tilts, is maximum near 1000 km, and has vanishing divergence near the equator. The planetary-scale component is again westerly, weaker than the synoptic-scale component and tightly equatorially confined as in the previous examples. This example is unrealistic since it drives mean westlies right at the equator and mean easterlies just poleward, yielding strong meridional wind shears. Nonetheless, it reemphasizes the conclusion that equatorial superrotation is significant in all the cases with vertically tilted superclusters irrespective of the meridional tilts and thus the meridional fluxes.

4. Discussion

a. Dynamical basis for upscale momentum flux and superrotation

We have shown that equatorial superrotation arises in all four MJO models irrespective of the synoptic-scale meridional tilt and, furthermore, that it is dominated by the first baroclinic component in all the cases where it is significant. It is natural to inquire as to which property of the planetary-scale flow causes this superrotation.

Let us restrict attention to the structure of the first baroclinic mode flow. Using the Helmholtz–Hodge–Weyl decomposition, we can express the horizontal velocity as

\[
U = -\Psi_y + \Phi_x, \\
V = \Psi_x + \Phi_y, 
\]

where \(\Psi\) expresses purely rotational motion in the plane, and \(\Phi\) describes horizontally divergent flow. Clearly the latter is due to the vertical component of the circulation since \(w_z = -\Phi_{xz} - \Phi_{yz}\) because of incompressibility. The flow is convergent at the base of the troposphere if \(w_z > 0\) yielding a potential that is negatively curved, for example, \(\Phi = \Phi_0 e^{-\left(x^2 + y^2\right)/2}\) with \(\Phi_0 > 0\). Considering flows that are symmetric about the origin further requires that \(\Psi\) be antisymmetric; \(\Psi = -\Psi_0 e^{-\left[(x-k)^2 + y^2\right]/2}\) with \(\Psi_0 > 0\) describes equatorially symmetric pair of cyclonic gyres, zonally shifted with respect to the potential flow.

It is a straightforward calculation to show that, for small values of the shift in this simple example, the zonal mean meridional momentum flux is

\[
\langle UV \rangle \approx -y\Psi_0\Phi_0 e^{-y^2/2} 
\]
near $y = 0$, which is to say that only the correlation of rotation and convergence contribute to this flux. Because of the assumed separability of the rotational and convergent components of the flow, the other terms in the fluxes due to rotational/rotational and convergent/ convergent velocity correlations vanish. Though this is not the most general flow possible, it does highlight an important fact: equatorial superrotation is a result of the correlation of convergent and vortical components of the flow in the horizontal plane. In particular, both cyclonic/lower troposphere convergent flow and anticyclonic/lower troposphere divergent flow drive equatorial westerlies.

The next question we pose is: what are the necessary features of the planetary-scale heating and upscale momentum flux that create equatorial superrotation? Since equatorial superrotation occurs even in the canonical model of vertically tilted superclusters and congestus heating (the first example), it is sufficient to consider zonal momentum forcing due to vertical upscale flux and planetary-scale heating alone: the upscale temperature flux in the canonical model is extremely small near the equator. Again, we focus on the first baroclinic mode since it dominates the upscale fluxes.

To get a closed form analytic expression for the upscale flux, we shall approximate the longwave Eqs. (2.5) using balanced dynamics. This is justified since the propagation speed of the convective envelope (and thus

![Image](image_url)
the forcing) is \(5 \text{ m s}^{-1}\), which means that \(s = 0.1\) in the nondimensionalization. The equilibrium solutions are attained by the replacement \(\partial_t \rightarrow -s \partial_x\), which can be disregarded when \(s\) is small. Furthermore, the dissipation times correspond to \(d_0 = 0.55\) and \(d_x = 0.18\). Therefore, the effect of dissipation can be considered as a higher order effect in a perturbation expansion, the lowest order of which corresponds to longwave balanced dynamics. In this limit, the zonal and meridional velocities are given by

\[
U = \int_{x_0}^{x} (-2S^\theta - yS^\theta_y + F^U_y) \, dx
\]

\[
V = yS^\theta - F^U_y, \tag{4.3}
\]

where \(x_0\) is the stagnation point of the zonal flow on the equator. This point occurs just east of the middle of the convective envelope (see Figs. 2, 5, 8, 11), but it suffices to put it at the center of envelope when considering balanced dynamics. Again, focusing on the first baroclinic mode, the direct heating is symmetric about the center of the envelope whereas the upscale momentum flux is antisymmetric and a simple form for these functions is

\[
S^\theta = S \cos(x/2)e^{-y^2/2},
\]

\[
F^U = -F \sin(x)e^{-y^2/2}. \tag{4.4}
\]

Therefore, when \(SF > 0\), the meridional momentum flux is convergent at the equator and drives a mean westerly wind. Clearly, Eq. (4.5) implies that if either the upscale momentum flux from the synoptic-scales or the planetary-scale mean heating vanish, then there is no meridional flux of zonal momentum near the equator. However, in the canonical MJO model (and all of the other models considered in this paper), the zonal momentum flux convergence at the base of the troposphere, \(F^U\), drives westerlies in the western half of the envelope and easterlies in the eastern half of the envelope. The planetary-scale mean heating forces a horizontal convergence at the base of the troposphere, also driving westerlies in the western half of the envelope and easterlies in the eastern half. Therefore, the flow associated with the zonal momentum and mean heating reinforce one another and this is the source of the superrotation. If the tilts of the synoptic-scale flows were opposite those of the canonical example then the momentum forcing would be easterly in the west and westerly in the east of the envelope. In this case \(F < 0\) and the flux convergence would drive easterly mean winds near the equator.

b. Observational verification

Momentum fluxes derived from observational data require cautious interpretation since these measurements are fraught with uncertainty. Fluxes are usually inferred from the residual of the other terms in the
momentum equations. The analysis of Tung and Yanai (2002) of multiscale observational data from the intensive observation period (IOP) of the Tropical Ocean Global Atmosphere Coupled Ocean–Atmosphere Response Experiment (TOGA COARE) estimates the convective momentum transport in the vertical direction regarding two MJO events that crossed the intensive flux array (IFA) during the IOP. The westerly wind anomaly in mid-to-low troposphere as well as the westward tilt with height occurs in their analysis. The upscale transfer of the zonal momentum flux in the vertical direction is less prominent but does occur episodically. Tung and Yanai suggest that the upscale flux may be underestimated since it tends to occur in squall lines that are not properly resolved by the IOP observations.

Reanalyses products have been used to infer the morphology of the MJO. A cautionary note is that the MJO is usually weak in global models, so reanalyses products may contain uncertainties that are difficult to quantify. Sperber (2003) used the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis. Westward tilts of MJO zonal and vertical velocities are evident and exhibit considerable variability (cf. our theoretical results). For example, the westward tilts are prominent in the west Pacific locale but less so in the Indian Ocean.

The superrotation properties of the MJO associated with the transport of zonal momentum examined herein center on purely atmospheric mechanisms. Ob-

Fig. 10. Case 4: horizontal velocity at different heights in the troposphere. Compare with Fig. 1.
observational superrotation investigations using reanalysis products have centered on torques at the atmosphere–surface interface, which is how the earth’s rotation is affected (Weickmann et al. 1997). Our dynamical models of the MJO and the accompanying momentum transport, which fundamentally affect the torques, may usefully guide future observational analyses as well as the alternative approach of interpreting synthetic data from global cloud-system resolving models. The latter may prove particularly fruitful noting that strong MJOs commonly occur in cloud-system resolving models. The latter may prove particularly fruitful noting that strong MJOs commonly occur in cloud-system resolving models.

In conclusion, we have derived the expression for upscale fluxes for the IPESD multiscale models of the MJO, particularly focusing on the meridional momentum flux. It is the zonal and planetary mean of the meridional momentum flux convergence, which drives superrotation; that is, mean westerly winds. Because of the synoptic-scale balance dynamics in a weak temperature gradient environment, we can show that, at the equator, superrotation due to the upscale flux from the synoptic scales must necessarily be zero. In a series of four examples with varying degrees of synoptic-scale meridional momentum flux, all yielding MJO-like flows, we have shown that the equatorial superrotation is insensitive to the structure of the synoptic-scale meridional momentum flux; equatorial superrotation occurs in all cases. The preceding discussion shows that equatorial superrotation arises because of the correlation of upscale vertical momentum flux and planetary scale mean heating in such manner that the convergence or divergence of the flow due to either forcing mechanism must reinforce the other in order to drive mean equatorial westerlies.

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