Limits of predictability in the North Pacific sector of a comprehensive climate model

Dimitrios Giannakis¹ and Andrew J. Majda¹

We study limits of interannual to decadal predictability of sea surface temperature (SST) in the North Pacific sector of the Community Climate System Model version 3 (CCSM3). Using a set of low-frequency and intermittent spatiotemporal SST modes acquired through nonlinear Laplacian spectral analysis (a nonlinear data manifold generalization of singular spectrum analysis), we build a hierarchy of regression models with external factors to determine which modes govern the dynamic evolution and predictability of prominent large-scale patterns, namely the Pacific Decadal Oscillation (PDO) and North Pacific Gyre Oscillation (NPGO). Retaining key triple correlations between prognostic variables and external factors, as well as the seasonality of the data, we find that the PDO and NPGO modes of CCSM3 can be described with remarkably high fidelity as an outcome of forcing by the intermittent modes (with phase demodulation by the seasonal cycle) and cubic interactions between the low-frequency modes. Our results differ from the classical picture of ENSO-driven autoregressive models for North Pacific SST variability, providing evidence that intermittent processes, such as variability of the Kuroshio current, limit long-range predictability in this climate model.

1. Introduction

A key question in the dynamics of the North Pacific ocean-atmosphere system is how much of the low-frequency (interannual to decadal) variability and long-range predictability of the sea surface temperature (SST) is due to processes internal to the extratropics, as opposed to external mechanisms coupled to the tropics. Some observational and numerical studies suggest that the dominant low-frequency mode in the North Pacific SST, namely the Pacific Decadal Oscillation (PDO), is strongly correlated to the El Niño-Southern Oscillation (ENSO) due to the atmospheric bridge [Alexander et al., 2002]. As result, in conventional scalar autoregressive (AR) models, the PDO is forced additively by both atmospheric noise and an ENSO external factor, giving rise to a reddened signal with interannual variability, both atmospheric noise and an ENSO external factor, giving rise to a reddened signal with interannual variability, but little predictability beyond those timescales [Newman et al., 2003]. In multivariate models, a mode known as North Pacific Gyre Oscillation (NPGO) [Di Lorenzo et al., 2008] or Victoria pattern [Bond et al., 2003], which is obtained through the second empirical orthogonal function (EOF) of SST and interpreted as the oceanic expression of the atmospheric North Pacific Oscillation, is introduced to describe aspects of low-frequency variability which cannot be explained by the PDO alone. Tropical forcings are thought to play a role in the dynamics of that mode as well.

In this work, we demonstrate that the prominent patterns for low-frequency SST variability in the North Pacific sector of a comprehensive climate model (the Community Climate System Model version 3 (CCSM3)) can be self-consistently described in terms of a set of spatiotemporal modes which carry two orders of magnitude less variance than the ENSO signal in the North Pacific, and yet can explain the conventional PDO and NPGO modes with pattern correlation higher than 90% in decadal-scale hindcasts. These patterns were recently identified using a data analysis algorithm called nonlinear Laplacian spectral analysis (NLSA) [Giannakis and Majda, 2012a, b], which generalizes classical singular spectrum analysis (SSA) [Aubry et al., 1991; Ghil et al., 2002] to take into account nonlinear manifold structure of datasets generated by complex dynamical systems. Characterized by temporal intermittency, these modes depend crucially on phase relationships with the annual cycle and its harmonics, and thus cannot be detected if the data is seasonally detrended.

The intermittent modes are associated with well-known features of the North Pacific, such as the boundary currents (Kuroshio, Oyashio, Alaska, and California currents), and exhibit strong triple correlations with the PDO and NPGO modes, despite being orthogonal to those patterns in both space and time. Owing to such correlations, they are used here as external factors in joint regression models of the PDO and NPGO, where cubic interactions also contribute to improved skill. Our study thus provides evidence that (at least in CCSM3) limits of predictability of large-scale modes like the PDO and NPGO can be reached through knowledge of features of the North Pacific associated with the intermittent modes, without direct reference to ENSO signals. Conversely, the low-frequency modes, suitably modulated by multiplicative couplings to the annual cycle, can explain most of the variability of the intermittent modes, suggesting that these two families of modes can potentially be used in self-contained stochastic models for North Pacific SST variability.

2. Spatiotemporal modes recovered by NLSA

The dataset used in this study is the monthly-averaged SST field extracted from a 500 y control integration of CCSM3 with a T85 atmosphere [experiment b30.009; see Collins et al., 2006] sampled on the model’s native ocean grid (1° nominal horizontal resolution) in the region 20°N-65°N and 120°E-110°W. This dataset was part of the cross-model comparative study by Giannakis and Majda [2012c] (dataset designation CS5), where more details of the data and NLSA algorithms can be found. Here, we note that of key importance was to analyze the data without annual cycle subtraction or partitioning into seasonal averages. As demonstrated in related work on depth-averaged data [Giannakis and Majda, 2012b], removing the monthly climatology distorts the nonlinear data manifold in a manner that eliminates the dynamically important intermittent modes, which are central to the regression framework developed here.
NLSA produces a set of temporal patterns \( v_j(t) \) (analogous to principal components) and a corresponding set of orthogonal spatial patterns \( u_k \) in lagged-embedding space (analogous to extended EOFs), which can be grouped in families of periodic, low-frequency, and intermittent modes. The \( v_j(t) \) patterns are orthogonal with respect to a weighted inner product associated with square-integrable \( L^2 \) spaces of scalar functions on the nonlinear data manifold [Gianakis and Majda, 2012a, b]. The regression models introduced below are based on various combinations of \( v_k(t) \), which are distinguished using the notations \( P_i(t) \), \( L_i(t) \), and \( I_i(t) \) for the \( i \)-th periodic, low-frequency and intermittent modes, respectively, in order of explained variance. Specifically, we examine the two-folded and three-folded set of annual modes \( \{P_1, P_2\} \) \( \leftrightarrow \{v_1, v_2\} \); the PDO, NPGO, and ENSO modes \( \{L_1, L_2, L_3\} \) \( \leftrightarrow \{v_3, v_4, v_5\} \); the leading four intermittent modes \( \{I_1, I_2, I_3, I_4\} \) \( \leftrightarrow \{v_6, v_7, v_12, v_13\} \). The time dependence and frequency spectra of these modes are shown in Figure 1; Animation 1 in the auxiliary material displays the dynamic evolution of the projected SST anomaly field onto the component spatial patterns \( u_k \) [see also Gianakis and Majda, 2012c]. Certain noteworthy features of the modes are as follows.

The annual periodic modes, \( \{P_1, P_2\} \), are sines and cosine waves evolving in temporal quadrature, and are not equivalent to monthly climatology [Gianakis and Majda, 2012b]. They provide crucial phase information for the interaction between the low-frequency and intermittent modes. \( L_1 \) and \( L_2 \) are the familiar PDO and NPGO patterns employed in bivariate indices for North Pacific SST variability [Bond et al., 2003; Di Lorenzo et al., 2008], characterized by significant power over interannual to decadal timescales and absence of significant spectral peaks. Mode \( L_3 \) is the manifestation of ENSO in the North Pacific sector in CCSM3, involving a coastal Kelvin pattern of the California coast; see [Mann and Emanuel, 2006]. It is a known issue with the behavior of ENSO in this model that the extratropical ocean is less strongly correlated to the tropics than in nature. Moreover, as manifested by the concentration of Fourier power in 2-4 y timescales, the model’s ENSO exhibits a degree of biennial quasi-periodicity which is not observed in the real ocean [Collins et al., 2006].

The intermittent modes come in near-degenerate pairs, \( \{I_1, I_2\} \) and \( \{I_3, I_4\} \), but do not evolve in exact quadrature. Prominent spatial features of \( P_1 \) and \( I_1 \) are SST anomalies in the North Pacific current (flowing eastward along \( \sim \text{40}^\circ \text{N} \)) and subtropical gyre accompanied with anomalies of opposite sign in the Alaska and California currents. Pair \( \{I_1, I_2\} \) is characterized by SST anomalies shed eastward from the coast of Japan associated with the Kuroshio-Oyashio extension region as well as activity in the subpolar gyre. Modes \( I_1-I_4 \) all have the temporal structure of modulated signals with a carrier frequency of \( 1 \text{ y}^{-1} \). This means that they can be demodulated to extract their low-frequency amplitude and phase components via linear combinations of products with the seasonal periodic modes of the form \( P_i(t)I_j(t) \).

The intermittent modes exhibit non-trivial higher-order lagged correlations; in particular, triple and quadruple correlations. We assess the significance of such correlations through the corresponding lagged correlation functions, \( \langle v_i(\tau)v_j(\tau + t_1)v_k(\tau + t_2) \rangle \) and \( \langle v_i(\tau)v_j(\tau + t_1)v_k(\tau + t_2)v_l(\tau + t_3) \rangle \), examples of which are shown in Figure 2. Here, angle brackets denote temporal averaging with respect to \( \tau \), so that the first argument in \( \langle v_i(\tau)v_j(\tau + t_1) \rangle \) is leading. Even through the PDO and NPGO modes exhibit negligible double correlations with the intermittent modes, rich lead–lag relationships emerge when triple correlations are considered. For instance, note the time asymmetry between \( \langle L_1P_1L_2 \rangle \) and \( \langle L_1P_2L_3 \rangle \). It is therefore natural to incorporate such products in regression models. Quadruple correlations (not shown here) are also significant for certain combinations of modes, suggesting that cubic interactions may also be important.

### 3. Regression methodology

Following the red noise paradigm of Frankignoul and Hasselmann [1977], the canonical autoregressive models for low-frequency SST variability in the North Pacific have the form

\[
L_i(t+1) = A_i^2L_i(t) + B_i^2P_i(t) + C_i^2[\mathbf{L}^* + \mathbf{L}]_i(t) + \sigma_i \epsilon(t). \tag{1}
\]

In the above, the \( i \)-th low-frequency mode at time \( t + 1 \) depends linearly on a vector \( L_i(t) \) of low-frequency modes at time \( t \) (also governed by prognostic equations similar to (1)), a vector of prescribed external factors \( P_i(t) \), and a scalar Gaussian process \( \epsilon(t) \). For instance, a model of this type for mode \( L_1 \) with \( L_1 = [L_1] \), \( F_1 = [L_3] \), and 1 y timestep corresponds to the ENSO-forced scalar regression model for the PDO of Newman et al. [2003]. Dropping \( F_1 \) leads to a standard red noise model for regime behavior in the North Pacific [Rudnick and Davis, 2003].

Here, in light of the more recent paradigm involving both the PDO and NPGO [Bond et al., 2003; Di Lorenzo et al., 2008], our baseline regression setup is a bivariate model for modes \( L_1 \) and \( L_2 \) with autoregressive components \( L_1 = [L_1, L_2] \) and, optionally, an ENSO external factor \( F_1 = [L_3] \). Note that the timestep in these models is 1 m rather than 1 y. Moreover, because the cross correlation \( \langle L_2(\tau)L_3(\tau + 1) \rangle \) is small at one-month lags, we do not include an ENSO external factor in the prognostic equation for \( L_2 \). We will return to the question of thresholding regression coefficients below.

Motivated by the triple correlation results of Figure 2 (as well as the corresponding quadruple correlations), we put forward an alternative model for the low-frequency SST modes:

\[
L_i(t+1) = A_i^2L_i(t) + B_i^2[\mathbf{P}^* + \mathbf{L}]_i(t) + C_i^2[\mathbf{L}^* + \mathbf{L}]_i(t) + \sigma_i \epsilon(t), \tag{2}
\]

In equation (2), the terms \( [\mathbf{P}^* + \mathbf{L}]_i(t) \) and \( \mathbf{L}^* + \mathbf{L} \) are vectors whose elements are given by double and triple products of the form \( P_j(t)L_k(t) \) and \( L_j(t)L_k(t)L_l(t) \), respectively. Thus, compared to the canonical case in equation (1), this class of models features (i) forcing of the low-frequency PDO and NPGO modes by the intermittent modes, suitably demodulated via products with the annual periodic modes; (ii) cubic interactions between the PDO and NPGO modes.

Potentially serious drawbacks of models with the structure of equation (2) is quartic increase of the number of regression coefficients \( C_i \) with the number of prognostic variables, as well as finite-time blowups [Majda and Yuan, 2012]. Here, we simultaneously sparsify and ensure model stability by retaining only a small number of double and triple product interactions exceeding a threshold of the corresponding correlations. Specifically, for some a priori set parameters \( \theta_B \) and \( \theta_C \), we retain only those interactions \( P_j(t)L_k(t) \) and \( L_j(t)L_k(t)L_l(t) \) in equation (2) meeting the conditions \( |\langle P_j(\tau)L_k(\tau)L_l(\tau + 1) \rangle| \geq \theta_B \) and \( |\langle L_j(\tau)L_k(\tau)L_l(\tau)(\tau + 1) \rangle| \geq \theta_C \). This simple yet effective approach is justified from the fact that empirical estimates of correlations enter in the ordinary least squares (OLS) equations [e.g., von Storch and Zwiers, 2002] for the estimation of \( B_i \) and \( C_i \), and sampling errors can adversely affect the conditioning of those equations when the true correlations are small. On the basis of similar threshold criteria we
have rejected the inclusion of a direct ENSO forcing in equation (1) for the NPGO mode, as well as a variety of nonlinear interactions, including quadratic interactions, \( L_j(t)L_k(t) \), multiplicative couplings between the low-frequency and intermittent modes, \( I_l(t)I_k(t) \), double products between intermittent modes, \( I_l(t)I_k(t) \), and cubic terms of the form \( L_j(t)L_k(t)I_l(t) \), \( L_j(t)L_k(t)I_l(t) \), and \( I_l(t)I_k(t)I_l(t) \).

Continuing this line of argument, we write down the corresponding model regression model for the intermittent modes,

\[
I(t+1) = A^I(t) + B^I(P + L)_I(t) + C^I(t + 1)_I(t) + \sigma(t),
\]

(3)

where the low-frequency modes are treated as external factors, and interactions are pruned by thresholding \(|(P_l(t)I_k(t))I_l(t+1)|\) and \(|(I_l(t)I_k(t))I_l(t)|\).

4. Results and discussion

We trained bivariate regression models for the PDO \( (L_1) \) and NPGO \( (L_2) \) modes as specified in equations (1) and (2) by applying OLS to the first half of the available time series for \( u(t) \) (a total of 2688 monthly samples), and used the second half to evaluate the pattern correlation score and RMS error of hindcasts made by those models and the prescribed external factors. Here, we present results obtained from a suite of five regression models, which are summarized in Table 1. Model 1 is a standard autoregressive model from equation (1) with no external factors, model 2 incorporates the ENSO mode \( (L_2) \) as an additive external factor, and models 3–5 are all forced by products of annual periodic and intermittent modes, in accordance with equation (2). The coefficients of model 3 were evaluated without including any of the linear \( (A^I) \) and cubic \( (C^I) \) terms. Model 4 augments model 3 by including \( A^I, B^I \), and both \( A^I \) and \( C^I \) are incorporated in model 5. In models 3–5, we selected the statistically significant \( P_lI_l \) and \( L_jL_kI_l \) interactions using threshold values \( \theta_B = 0.2 \) and \( \theta_C = 0.8 \), resulting in a total of six forcings from the intermittent modes and two cubic interactions per low-frequency mode.

Using the same portion of the available data for training, we also built four models for the intermittent modes based on equation (3). In this case, models 1 and 2 contain only \( A^I, B^I \), and terms, respectively, model 3 contains both \( A^I \) and \( B^I \), and model 4 features all of \( A^I, B^I, C^I \), and \( C^I \) coefficients. The correlation threshold parameters were set to the same \( \theta_B = 0.2 \) and \( \theta_C = 0.8 \) values as above. Because the models for the intermittent modes are four-dimensional and thus feature a larger number of coefficients than their low-frequency counterparts, we have placed a listing of the coefficient values in Tables S1–S4 in the auxiliary material in the interest of brevity. Example trajectories and skill scores for hindcasts made using these models are displayed in Figures 3 and 4.

First, we turn to the canonical autoregressive models for low-frequency variability (models 1 and 2 in Table 1). These models fail to track the PDO and NPGO on average beyond two and five years, respectively, with the ENSO-based external factor model achieving a moderate gain of 15–20% in residual pattern correlation score. In contrast, all of the models forced by products of the periodic and intermittent modes are able to track the truth signal with pattern correlation exceeding 90%. The models which do not permit any interaction between the prognostic variables are generally less skillful than those with linear or cubic terms. This is especially obvious in Figure 4, where adding linear couplings between the intermittent modes leads to a significant improvement of both the pattern correlation and RMS error scores. On the other hand, the cubic terms produce negligible improvement of skill in these models. However, nonlinear terms are important for the PDO mode in Figure 3, where the RMS error of the cubic model is smaller by approximately a factor of two relative to the linear case. In the case of the NPGO mode, an overdamping observed in the model with linear but not cubic terms is alleviated by including the cubic interaction, but the overall improvement of skill compared to the non-interacting case is negligible.

In summary, the main result of this analysis is that the forcing by the intermittent modes, demodulated by the annual periodic modes, is able to reproduce the timeseries for the PDO and NPGO modes in CCSM3 with remarkable skill. Despite the fact that these modes are orthogonal in both lagged embedding space (through the corresponding spatial patterns \( u(t) \)) and time (with respect to the weighted inner product in NLSA). This picture appears to be at odds with the canonical models of low-frequency variability of North Pacific SST in equation (1), where predictability of low-frequency variables is governed by ENSO signals evolving at interannual timescales and the decorrelation time of a red noise process.

In this work, the variability of the PDO and NPGO patterns is closely tied to forcing from modes associated with the boundary currents and the subpolar and subtropical gyres, suitably phased by the seasonal cycle. Conversely, we have demonstrated that the low-frequency modes are highly significant external factors in tracking the behavior of the intermittent modes. As a result, in the North Pacific sector of CCSM3, the low-frequency and intermittent sets of patterns extracted by NLSA behave in a complementary manner, limiting each other’s predictability. The general approach developed here should be useful in finding the limits of predictability in other complex models with other forecasting scenarios.

It is important to note we have not constructed such a scheme for self-contained forecasts. That would involve time-advancing the governing equations for \( L_1 \) and \( L_1 \) in tandem, which may require additional interactions or prognostic variables if the model is to achieve skill beyond standard AR models. To that end, a potentially fruitful avenue of future research would be to apply statistical methods developed in the context of Bayesian spatiotemporal modeling [Wilk and Holan, 2011, and references therein] to inform the structure of self-contained forecast models.

Acknowledgments. This work was supported by NSF grant DMS-0456713 and ONR DRI grants N00014-10-1-0554.

References


Table 1. Interaction coefficients and noise amplitude of the bivariate regression models for the PDO and NPGO modes ($L_1$ and $L_2$) from equations (1) and (2). ✓ (X) indicates external factors/interactions which were (were not) part of a given model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Ext. factors</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$, $P_2$</td>
<td>X</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>$L_3$</td>
<td>X</td>
<td>✓</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>$I_1, . . . , I_4$</td>
<td>X</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mode $L_1$</td>
<td>$A_1(L_1)$</td>
<td>0.9964</td>
<td>0.9972</td>
<td>X</td>
<td>0.6233</td>
<td>−0.0067</td>
</tr>
<tr>
<td></td>
<td>$A_1(L_2)$</td>
<td>0.0343</td>
<td>0.0330</td>
<td>X</td>
<td>−0.0624</td>
<td>−0.2148</td>
</tr>
<tr>
<td></td>
<td>$B_1(L_3)$</td>
<td>X</td>
<td>−0.0340</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>$B_1(P_1, I_1)$</td>
<td>X</td>
<td>X</td>
<td>0.4070</td>
<td>0.1732</td>
<td>0.4606</td>
</tr>
<tr>
<td></td>
<td>$B_1(P_2, I_2)$</td>
<td>X</td>
<td>X</td>
<td>0.4762</td>
<td>0.1954</td>
<td>0.5241</td>
</tr>
<tr>
<td></td>
<td>$B_1(P_1, I_3)$</td>
<td>X</td>
<td>X</td>
<td>0.2709</td>
<td>0.0907</td>
<td>0.2508</td>
</tr>
<tr>
<td></td>
<td>$B_1(P_2, I_3)$</td>
<td>X</td>
<td>X</td>
<td>−0.1778</td>
<td>−0.0180</td>
<td>−0.0598</td>
</tr>
<tr>
<td></td>
<td>$C_1(L_1, L_1, L_1)$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>−0.0032</td>
</tr>
<tr>
<td></td>
<td>$C_1(L_1, L_2, L_2)$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>0.0046</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td></td>
<td>0.0735</td>
<td>0.0658</td>
<td>0.1582</td>
<td>0.0696</td>
<td>0.0800</td>
</tr>
<tr>
<td>Mode $L_2$</td>
<td>$A_2(L_1)$</td>
<td>−0.0282</td>
<td>−0.0282</td>
<td>X</td>
<td>−0.0303</td>
<td>−0.1787</td>
</tr>
<tr>
<td></td>
<td>$A_2(L_2)$</td>
<td>0.9916</td>
<td>0.9916</td>
<td>X</td>
<td>0.9451</td>
<td>−0.0107</td>
</tr>
<tr>
<td></td>
<td>$B_2(P_1, I_1)$</td>
<td>X</td>
<td>X</td>
<td>0.1172</td>
<td>0.0001</td>
<td>0.2144</td>
</tr>
<tr>
<td></td>
<td>$B_2(P_2, I_1)$</td>
<td>X</td>
<td>X</td>
<td>−0.2610</td>
<td>−0.0409</td>
<td>−0.2397</td>
</tr>
<tr>
<td></td>
<td>$B_2(P_1, I_2)$</td>
<td>X</td>
<td>X</td>
<td>0.1604</td>
<td>0.0196</td>
<td>0.2418</td>
</tr>
<tr>
<td></td>
<td>$B_2(P_2, I_2)$</td>
<td>X</td>
<td>X</td>
<td>0.4128</td>
<td>0.0540</td>
<td>0.3653</td>
</tr>
<tr>
<td></td>
<td>$B_2(P_1, I_3)$</td>
<td>X</td>
<td>X</td>
<td>−0.4850</td>
<td>0.0128</td>
<td>−0.4414</td>
</tr>
<tr>
<td></td>
<td>$B_2(P_2, I_3)$</td>
<td>X</td>
<td>X</td>
<td>0.0118</td>
<td>0.0275</td>
<td>−0.0358</td>
</tr>
<tr>
<td></td>
<td>$C_2(L_1, L_1, L_2)$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>0.0141</td>
</tr>
<tr>
<td></td>
<td>$C_2(L_2, L_2, L_2)$</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>0.0048</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td></td>
<td>0.1330</td>
<td>0.1330</td>
<td>0.1590</td>
<td>0.1167</td>
<td>0.1573</td>
</tr>
</tbody>
</table>


Dimitrios Giannakis, Courant Institute of Mathematical Sciences, New York University, 251 Mercer St, New York, NY 10012, USA. (dimitris@cims.nyu.edu)

Andrew J. Majda, Courant Institute of Mathematical Sciences, New York University, 251 Mercer St, New York, NY 10012, USA.
Figure 1. Example trajectories, shown for years 420–470 of CCSM3 experiment b30.009, and Fourier spectra of the temporal modes $v_k(t)$ used for regression modeling.

Figure 2. Selected triple correlations $\langle v_i(\tau) v_j(\tau + t_1) v_k(\tau + t_2) \rangle$ of the temporal modes from NLSA.
Figure 3. Sample trajectories and skill scores for the regression models for the PDO and NPGO from equations (1) and (2). Results from models 1–5 of Table 1 are plotted in cyan, red, magenta, green, and blue, respectively. Black lines correspond to the truth signal. Note the enhanced capability of tracking the PDO and NPGO signals by including the intermittent modes as external factors (magenta, green, and blue lines).

Figure 4. Example trajectories and pattern correlation scores for the regression models for the intermittent modes from equation (3). Results from models 1–4 of Tables S1–S4 are plotted in red, magenta, green, and blue, respectively. Black lines correspond to the truth signal.