Using the stochastic multicloud model to improve tropical convective parameterization: A paradigm example

YEVGENIY FRENKEL

Department of Mathematics, and Center for Atmosphere-Ocean Science,

Courant Institute, New York University, New York, New York

ANDREW J. MAJDA

Department of Mathematics, and Center for Atmosphere-Ocean Science,

Courant Institute, New York University, New York, New York

BOUALEM KHOUIDER *

Department of Mathematics and Statistics, University of Victoria, Victoria, BC, Canada

*Corresponding author address: Boualem Khouider, Department of Mathematics and Statistics, University of Victoria, 3800 Finnerty Road, Victoria, British Columbia, Canada.
E-mail: khouider@uvic.ca
ABSTRACT

Despite recent advances in supercomputing, current general circulation models (GCMs) poorly represent the variability associated with organized tropical convection. A stochastic multicloud convective parameterization based on three cloud types (congestus, deep, and stratiform), introduced recently by Khouider, Biello and Majda in the context of a single column model, is used here to study flows above the equator without rotation effects. The stochastic model dramatically improves the variability of tropical convection compared to the conventional moderate and coarse resolution paradigm GCM parameterizations. This increase in variability comes from intermittent coherent structures such as synoptic and mesoscale convective systems, analogs of squall lines and convectively coupled waves seen in nature whose representation is improved by the stochastic parameterization. Furthermore, simulations with sea surface temperature (SST) gradient yield realistic mean Walker-cell circulation with plausible high variability. An additional feature of the present stochastic parameterization is a natural scaling of the model from moderate to coarse grids which preserves the variability and statistical structure of the coherent features. These results systematically illustrate, in a paradigm model, the benefits of using the stochastic multicloud framework to improve deterministic parameterizations with clear deficiencies.

1. Introduction

Organized convection in the tropics involves a hierarchy of temporal and spatial scales ranging from short lived mesoscale organized squall lines on the order of hundreds of kilometers to intraseasonal oscillations over planetary scales on the order of 40, 000 km (Nakazawa
1974; Hendon and Liebmann 1994; Wheeler and Kiladis 1999). Despite the continued research efforts by the climate community, the present coarse resolution GCMs, used for the prediction of weather and climate, poorly represent variability associated with tropical convection (Slingo et al. 1996; Moncrieff and Klinker 1997; Scinocca and McFarlane 2004; Lau and Waliser 2005; Zhang 2005). It is believed that the deficiency is due to inadequate treatment of cumulus convection (Moncrieff and Klinker 1997; Lin et al. 2006). Given the importance of the tropics for short-term climate and medium to long range weather prediction, the search for new strategies for parameterizing the unresolved effects of tropical convection has been the focus of researchers during the last few decades.

The search for methods of adequately addressing the interactions across temporal and spatial scales between the large scale circulation and organized cloud systems, from individual clouds to large-scale clusters and superclusters to planetary-scale disturbance, has not been fruitless. Cloud-resolving models on fine computational grids and high-resolution numerical weather prediction models with improved convective parametrizations succeed in representing some aspects of organized convection (ECMWF 2003; Moncrieff et al. 2007). However, due to their extremely high computational cost, these methods cannot be applied to large ensemble-size weather prediction or climate simulations. The complexity of the problem has motivated the development of an approach that directly addresses the multiscale nature of the problem. Superparameterization (SP) methods (Grabowski and Smolarkiewicz 1999; Grabowski 2001, 2004; Randall et al. 2003; Majda 2007) use a cloud resolving model (CRM) in each column of the large scale GCM to explicitly represent small scale and mesoscale processes and interactions among them. The computational cost can be further reduced by techniques such as sparse space-time SP (Xing et al. 2009). Nonetheless computationally
inexpensive GCM parameterizations that capture the variability and coherent structure of deep convection have remained a central unsolved problem in the atmospheric community.

The most common conventional cumulus parameterizations are based on the quasi-equilibrium (QE) assumption first postulated by Arakawa and Schubert (1974), the moist convective adjustment idea of Manabe et al. (1965), or the large scale moisture convergence closure of Kuo (1974) type. As such, the mean response of unresolved modes on large/resolved scale variables is formulated according to a prescribed deterministic closure. While many recipes for the closures have been created (Kain and Fritsch 1990; Betts and Miller 1986; Zhang and McFarlane 1995), these purely deterministic parameterizations were found to be inadequate for the representation of the highly intermittent and organized tropical convection (Palmer 2001). Many of the improvements in GCMs of the last decade came from the relaxation of the QE assumption, for example, through the addition of a stochastic perturbation. Buizza et al. (1999) used a stochastic backscattering model to represent the model uncertainties in a GCM while Lin and Neelin (2003) used a stochastic parametrization to randomize the way in which deep convection responds to large fluctuations via a prescribed probability distribution function for the convective time scale. Majda and Khouider (2002) were the first to propose a stochastic model for convective inhibition (CIN), that allows both internal interactions between convective elements and two-way interactions between the convective elements and the large scale/resolved variables. Their model is based on an Ising-type spin-flip model used as a model for phase transitions in material science (Katsoulakis et al. 2003). When coupled to a toy GCM, this stochastic parameterization produced eastward propagating convectively coupled waves that qualitatively resemble observations (Khouider et al. 2003; Majda et al. 2008) despite the extreme simplicity of the model and deficiency of
the underlying convective parameterization. Stochastic processes have been used to parameterize convective momentum transport (Majda and Stechmann 2008), to improve conceptual understanding of the transition to deep convection through critical values of column water vapor (Stechmann and Neelin 2010), as well as for the analysis of cloud cover data in the tropics and the extratropics (Horenko 2010).

The stochastic multicloud model for tropical convection introduced by Khouider et al. (2010) (hereafter KBM10) is a novel approach to the problem of missing tropical variability in GCMs. The stochastic parameterization is based on a Markov chain lattice model where each lattice site is either occupied by a cloud of a certain type (congestus, deep or stratiform) or it is a clear sky site. The convective elements interact with the large scale environment and with each other through convectively available potential energy (CAPE) and middle troposphere dryness. When local interactions between the lattice sites are ignored, a coarse grained stochastic process that is intermediate between the microscopic dynamics and the mean field equations (Katsoulakis et al. 2003; Khouider et al. 2003; Majda et al. 2008) is derived for the dynamical evolution of the cloud area fractions. Besides deep convection, the stochastic multicloud model includes both low-level moisture preconditioning through congestus clouds and the direct effect of stratiform clouds including downdrafts which cool and dry the boundary layer. The design principles of the multicloud parameterization framework are extensively explored in the deterministic version of the model developed by Khouider and Majda (2006a; 2006b; 2007; 2008a; 2008b, hereafter KM06a, KM06b,KM07, KM08a, KM08b, respectively).

Here a slightly modified version of the stochastic multicloud model, coupled to simplified primitive equations, with vertical resolution reduced to the first two baroclinic modes, is used
to study flows above the equator without rotation effects. The major modifications to the basic stochastic multicloud model (KBM10) include: direct dependence of congestus cloud cover on low level CAPE, a simplified stratiform heating closure and inclusion of stratiform rain in the precipitation budget. Additionally, minor changes in parameters, motivated in part by the single column sensitivity studies in KBM10, are introduced to further improve the intermittency of the solutions. The single column analysis of KBM10 is extended to flows above the equatorial ring. In this paradigm setting, we illustrate the advantages of the stochastic model through side by side comparison with deterministic parameterizations with clear deficiencies. Additional emphasis is placed on natural adaptability of the model to moderate and coarse GCM resolutions.

A brief review of the most salient features of the stochastic multicloud model along with major modifications in closures is presented in Section 2. The section also introduces a benchmark deterministic parameterization with clear deficiencies used here to illustrate the advantages of stochastic parameterization. In Section 3, single column simulation results are used to validate the changes in the tuning parameters of the stochastic parameterization. Additionally, the section contains a study of sensitivity of the new parameterization (in single column mode) to changes in the SST which here is deemed the “mock” warm pool. In Section 4, the parameterization is used to study flows above the equator on a moderate size GCM grid (40 km) in both aquaplanet and SST gradient regimes. Meanwhile, Section 5 presents a natural scaling principle for coarse grid resolution (160 km) simulations that retains the variability and coherent wave features of the moderate grid simulations. Some discussion and concluding remarks are given in Section 6.
2. Details of the model

In this section we briefly review the dynamical and physical features of the stochastic multicloud parameterization. The goal of this review is to highlight aspects of the model relevant to the discussion in the following sections and introduce modifications to the closures; thus only the most salient features of the multicloud modeling framework are presented here. A more complete discussion of the stochastic model and multicloud framework is found in the original papers (KBM10, KM06, KM08).

a. Dynamical core of the multicloud model

The multicloud parametrization framework assumes three heating profiles associated with the main cloud types that characterize organized tropical convective systems (Johnson et al. 1999): cumulus congestus clouds that heat the lower troposphere and cool the upper troposphere, through radiation and detrainment, deep convective towers that heat the whole tropospheric depth, and the associated lagging-stratiform anvils that heat the upper troposphere and cool the lower troposphere, due to evaporation of stratiform rain. Accordingly, Khouider and Majda (e.g. KM06a, KM08a) used the momentum and potential temperature equations for the first and second baroclinic modes of vertical structure, that are directly forced by deep convection and both congestus and stratiform clouds, respectively, as a minimal dynamical core that captures the main (linear response) effects of these three cloud types. Versions of this simple modeling framework that include effects of convective momentum transport (CMT) are found in Majda and Stechmann (2008, 2009) and Khouider et al. (2011). The multicloud model also carries equations for the vertically averaged moisture
(water vapor mixing ratio), over the tropospheric depth, and bulk boundary layer dynamics averaged over the atmospheric boundary layer (ABL).

For convenience, the governing equations, along the equatorial ring neglecting the effect of rotation are reported in Tables 1 and 2 while the associated constants and parameters are reported in Table 3. All equations are given in the non-dimensional form where the speed of the first baroclinic Kelvin waves, \( c_r \approx 50 \text{ ms}^{-1} \), is the velocity scale; the equatorial Rossby radius of deformation, \( L_e \approx 1500 \text{ km} \), is the length scale; \( T = L_e/c_r \approx 8.33 \text{ hr} \) is the time scale; and \( \bar{\alpha} = H_T N^2 \theta_0/\pi g \approx 15 \text{ K} \) is the temperature unit scale.

b. The stochastic multicloud model

The stochastic multicloud parameterization is designed to capture the dynamical interactions between the three cloud types that characterize organized tropical convection and the environment using a coarse grained lattice model (KBM10). To mimic the behavior within a typical GCM grid box, a rectangular \( n \times n \) lattice is considered, where each element can be either occupied by a congestus, deep or a stratiform cloud or is clear sky, through an order parameter that takes values of 0, 1, 2 or 3 on each lattice site. A continuous time stochastic process is then defined by allowing the transitions, for individual cloud sites, from one state to another according to intuitive probability transition rates, which depend on the large scale-resolved variables. These large scale variables are the convectively available potential energy integrated over the whole troposphere (CAPE), the convectively available energy integrated over the lower troposphere \( \text{CAPE}_l \) (see Table 2), and the dryness of the mid troposphere, which is a function of the difference between the atmospheric boundary layer (ABL) temper-
ature $\theta_{eb}$ and the middle tropospheric potential temperature $\theta_{em} = q + (2\sqrt{2}/\pi)(\theta_1 + \alpha_2 \theta_2)$.

The inclusion of dryness of the middle troposphere accounts for mixing of the convective parcels with dry environmental air (KM06a, KM06b, KM07, KM08a, KM08b, KBM10).

The probability rates are constrained by a set of intuitive rules which are based on observations of cloud dynamics in the tropics (e.g., Johnson et al. 1999; Mapes 2000, Khouider and Majda 2006, and references therein). Following KBM10, a clear site turns into a congestus site with high probability if low level CAPE is positive and the middle troposphere is dry. A congestus or clear sky site turns into a deep convective site with high probability if CAPE is positive and the middle troposphere is moist. A deep convective site turns into a stratiform site with high probability. Finally, all three cloud types decay naturally to clear sky at some fixed rate. All other transitions are assumed to have negligible probability. These rules are formalized in Table 4 in terms of the transition rates $R_{ik}$ and the associated time scales $\tau_{ik}$.

The effects of CAPE and dryness enter the model with scaling parameters $CAPE_0$ and $T_0$ whose values are specified in Table 3.

Notice that the assumption that the transition rates depend on the large scale variables accounts for the feedback of the large scales on the stochastic model, while ignoring the interactions between the lattice sites altogether implies that the stochastic processes associated with the different sites are identical. The latter simplification makes it easy to derive the stochastic dynamics for the GCM grid box cloud coverages alone, which can be evolved without the detailed knowledge of the micro-state configuration, by using a coarse-graining technique (Katsoulakis et al. 2003,b, Khouider et al. 2003) that yields here a system of three birth-death-like processes, corresponding to the three cloud types. The resulting birth-death Markov system is easily evolved in time using Gillespie’s exact algorithm (Gillespie 1975,
1977). Thus given the large scale thermodynamic quantities the stochastic process yields the dynamical evolution for the congestus, deep and stratiform cloud fractions $\sigma_c, \sigma_d$ and $\sigma_s$ respectively.

The heating associated with each cloud type is assumed to be proportional to the cloud fraction according to the closure equations (KBM10).

\begin{align*}
H_c &= \sigma_c \frac{\bar{\alpha}_c}{H_m} \sqrt{\text{CAPE}^+} \\
H_d &= [\bar{\sigma}_d \bar{Q} + \frac{1}{\tau_c(\sigma_d)} (a_1 \theta_{eb} + a_2 q - a_0 (\theta_1 + \gamma_2 \theta_2))]^+ \\
\tau_c(\sigma_d) &= \frac{\sigma_d}{\sigma_c} \tau_c^0 \\
H_s &= \alpha_s [\bar{\sigma}_s \bar{Q} + \frac{1}{\tau_c(\sigma_s)} (a_1 \theta_{eb} + a_2 q - a_0 (\theta_1 + \gamma_2 \theta_2))]^+ \\
\tau_c(\sigma_s) &= \frac{\sigma_s}{\sigma_c} \tau_c^0
\end{align*}

Here $\sigma_c, \sigma_d, \sigma_s$ denote the mean (longtime average) values of the congestus, deep and stratiform cloud area fractions.

From equation 1, we note that congestus heating $H_c$ is proportional to the product of the congestus cloud fraction and square root of low level CAPE. Thus both the congestus heating and congestus cloud fraction are tied to convectively available energy in the lower tropospheric region where congestus clouds are most active. Consistent with Lin and Neelin (2000) (see also Lin and Neelin 2003), the deep convective time scale $\tau_c$ is inversely proportional to the stochastic area fraction of deep convection. At equilibrium, when $\sigma_d = \bar{\sigma}_d$, $\tau_c$ is set to the value $\tau_c^0 = 2$ hours used in KBM10 and the original deterministic multicloud model of Khouider and Majda (2006a, 2008a). The deep convective heating closure, stated
in equations 2 and 3, depends on the boundary layer temperature $\theta_{eb}$, the averaged atmospheric moisture $q$ and the mid-tropospheric potential temperature $\theta_1 + \gamma_2 \theta_2$, through the coefficients $a_1$, $a_2$ and $a_0$. The first two parameters satisfy $(a_1 + a_2 = 1)$ and serve as a switch between CAPE and Betts-Miller parameterization, while the last parameter $a_0$ parametrizes the convective response to fluctuations in the dry buoyancy (KM06). Accordingly, the dry convective buoyancy parameter is set to a low value of $a_0 = 2$ relative to the values previously used in KBM10 to enhance deep convection. A new feature of the model formulation is the stratiform heating closure which takes the same form as deep convective closure. This new formulation combined with fixed transition rates for the formation (from deep convective clouds) and decay of stratiform clouds simplifies the model’s complexity.

While precipitation in KBM10 was solely due to deep convection, the multicloud framework allows for easy inclusion of congestus and stratiform rain (KM08a) into the precipitation budget. The precipitation influences the dynamics of the model through the first baroclinic heating mode through the term $H_d + \xi_c H_c + \xi_s H_s$, where $\xi_c$ and $\xi_s$ measure relative contributions of congestus and stratiform precipitation, respectively. Since the amount of congestus rain in the tropical ocean is on the order of 10 percent (Lin et al. 2002), the parameter $\xi_c$ is set to zero. At radiative convective equilibrium, the total precipitation heating is balanced by the imposed radiative cooling of 1 K/day. Due to the fact that stratiform creation and dissipation rate is fixed relative to the deep convective cloud cover, it is easy to derive a closed expression for averaged stratiform rain fraction $f_s$ at RCE by invoking the mean value theorem (as shown in KM08a).
\[ f_s = \frac{\xi_s \alpha_s^{\tau_{23}}}{1 + \xi_s \alpha_s^{\tau_{23}}} \]  \hspace{1cm} (6)

For the simulations presented here, the stratiform rain parameter \( \xi_s \) is chosen so that the model yields 40 percent stratiform rain in accordance with observations (Schumacher and Houze 2003).

Aside from the changes in closures noted above, a small number of changes in tuning parameters (compared to KBM10), mainly aimed at improving the intermittency of the stochastic parameterization, will be considered in Section 3 where the intermittent single column solutions are discussed and compared to previous results. These tuning parameter changes include CAPE and dryness adjustment parameters, the number of convective elements within one large scale grid box as well as certain changes to the cloud transition time scales.

c. A paradigm model deterministic GCM convective parameterization with clear deficiencies.

The design principles of the deterministic multicloud parameterization are based on the interactions of the same three cloud types. Its dynamical features and its capabilities, in terms of the impact of organized convection on the large scale tropical circulation and convectively coupled waves are demonstrated in Khouider and Majda (KM06a, KM06b,KM07, KM08a, KM08b), using both linear analysis and non-linear simulations. The deterministic closure takes into account the energy available for congestus and deep convection (respectively \( \bar{Q}_c \) and \( \bar{Q}_d \) see Table 2) and uses a non-linear moisture switch that allows natural transitions between congestus and deep convection. In the appropriate parameter regime,
the deterministic multicloud model is very successful in capturing most of the Wheeler-
Kiladis-Takayabu spectrum of convectively coupled waves (Takayabu 1994; Wheeler and
Kiladis 1999) in terms of linear wave theory (KM06a;KM08b; Han and Khouider 2010)
and nonlinear organization of large-scale envelopes mimicking across-scale interactions of
the Madden-Julian oscillation (MJO) and convectively coupled waves (KM07; KM08a), in
the idealized context of a simple two-baroclinic modes model employed here. Furthermore,
the parameterization has been used in the next generation of the National Center for At-
mospheric Research GCM (HOMME) and is very successful in simulating the MJO and
convectively coupled equatorial waves, at a coarse resolution of 170 km, in the idealized case
of a uniform SST (aquaplanet) setting (Khouider et al. 2011). However, the sensitivity of
the deterministic multicloud model to the key parameters means that in some physically
motivated parameter regimes (such as the physical regime used for the stochastic parame-
terization in this study) the model can perform poorly. Here such a suboptimal choice of the
parameters is taken to provide a reference point that roughly corresponds to the behavior of
a paradigm GCM parameterization with clear deficiencies.

The key difference between this simulation and the well tuned simulations of KM08a
and KM08b, lies in two parameter changes. First, the stratiform rain fraction is increased
from 0.1 to 0.4 in accordance with observations (Schumacher and Houze 2003). The higher
stratiform rain fraction increases the stability of the waves in the simulation and results
in reduced variability. Second, we use a relatively lower convective buoyancy parameter $a_0$
to increases strength of the instability; positive potential temperature anomalies become
less effective at impeding deep convection. Combined together the two parameter changes
produce simulations where the variability is reduced to convectively coupled waves of the

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type shown in Figure 2. These extremely stable waves propagate over the length of the
domain at 22 m/s in both homogeneous SST (KM08,KM06b) and warm pool scenarios. A
closer look at the structure of the waves reveals a convectively coupled wave with sharp
deep convective peak and surprisingly little congestus heating. In fact, due to the low dry
convective buoyancy frequency unimpeded deep convection dominates congestus heating.
The lack of congestus heating leads to vanishing of the heating field tilt that can be observed
in nature and the deterministic multicloud model in an optimal parameter regime.

In Figure 2, simulations on the domain with SST gradient (KM08) reveal another unde-
sirable feature of the detuned deterministic convective parameterization. The main feature
of the Walker circulation is a sharp peak in the congestus and deep heating near the center
of the warm pool. The sharp transition is characteristic of deterministic parameterization in
poor parameter regimes. As seen in Figure 3, convectively coupled waves travel the length
of the domain and interact only weakly with the warm pool.

While the model produces nontrivial amount of variability, the structure of both the mean
circulation and waves is not physical. Overall this illustrates one of the key weaknesses of
conventional deterministic parameterizations used in operational GCMs. In this suboptimal
regime, the model produces unphysical Walker circulation with weak variability about the
mean. Furthermore, the variability in the simulation comes from extremely stable wave
forms with unnatural structure. All of the above shortcomings will be corrected by the
stochastic parameterization considered in the following sections.
3. **Column Model validation**

In this section, we present some new results of the stochastic multicloud model in the context of a single column simulations (KBM10). As such, the section is used to explain typical behavior of the stochastic multicloud parameterization in the absence of spatial variations and clarify the changes in some basic tuning parameters and time scales compared to KBM10. The second goal of the section is to study the sensitivity of the single column model to SST variations, which will be contrasted against SST gradient simulations with full scale spatial effects presented in the next section.

The single column equations are obtained by disregarding spatial dependence components and the zonal wind in Table 1. As in KBM10, we employ a third order Adams-Bashforth method to integrate the resulting ODE system. The coarse grained birth-death process is evolved in time by means of an acceptance-rejection Markov Chain Monte Carlo method based on Gillespie’s exact algorithm (Gillespie 1975, 1977). The construction of such algorithm as well as the associated reduction in computational cost is discussed thoroughly in KBM10.

a. **Single Column simulations**

A time series of a typical solution for the parameter regime described in Table 3, is plotted in Figure 4. The top two panels present the plots of the large scale variables $\theta_1, \theta_2, \theta_{eb}$ and $q$. The middle two plots show the cloud fractions and associated heating profiles for the three cloud types while the bottom two panels show the total precipitation and large scale quantities that drive the stochastic dynamics, namely $C_1, C$ and $D$ (Table 3). The
most notable feature is the time synchronization of the oscillations of the stochastic and
deterministic variables which leads to time series with frequent precipitation peaks of 10
K/Day and more intermittent large precipitation events on the order of 25 K/Day. For
both the cloud fractions and resulting heating profiles, congestus bursts are followed by
deep convective bursts, which in turn lead to stratiform peaks, consistent with the physical
intuition utilized to design the model. The fluctuations in the cloud area fractions are
directly related to changes in the large scale fields. The increase in congestus area fraction
is a direct response to low level CAPE build up (corresponding to the $\theta_{eb}$-peaks). Congestus
heating anomalies then yield a rise in $\theta_2$ by direct heating, which yields a rise in $\theta_{em}$. The
moist atmosphere combined with high abundance of CAPE (which is not as sensitive to the
second baroclinic mode heating as low level CAPE) triggers the peak in $\sigma_d$ and associated
deep convective heating. This is followed by trailing stratiform clouds, since there is a non-
zero probability that a significant fraction of the deep convective clouds are converted into
stratiform clouds.

The main difference between the single column results reported here and the original
KBM10 publication is the highly intermittent nature of the large precipitation events. The
medium scale convective events are frequent but not periodic, while the larger convective
events are even more intermittent with standard deviation of the passage time equal to the
mean passage time (indicative of Poisson distribution in time). The relationship between
medium and large precipitation events is reminiscent of progressive deepening of the con-
vection on multiple scales (Mapes et al. 2006). Small congestus cloud peaks are followed
by small deep convective precipitation, driven primarily by congestus to deep conversion
(Figure 5). Deep convection is in turn followed by stratiform anvils, which cool and dry the
boundary layer by downdrafts, thus reduce CAPE, and at the same time help moisten the lower troposphere for the next convective episode, via the evaporation of stratiform rain. These small scale convective events are followed by larger congestus events, which moisten the environment to allow direct transition from clear sky to deep convective clouds, which account for a large percentage of the precipitation in the intermittent large scale convective events.

This intermittence is due in part to the systematic choice of the tuning parameters and transition time scales motivated in part by the sensitivity studies in KBM10. The first factor contributing to the variability is the relatively low number of convective elements set to $N = 30 \times 30$. It is easy to see that as the number of elements increases the dynamics of the stochastic process converges to deterministic mean field equations behavior losing some of the inherent intermittency of the stochastic process. Secondly, the choice of the $CAPE_0$ value, which can be viewed as an “activation energy”, is motivated by the observation that while large values of the threshold lead to large fluctuations in both the cloud area fractions and the large scale climate variables, small values of the activation energy lead to intermittent large events on top of very weak and fast oscillations. The value of $CAPE_0 = 200$ J/Kg strikes a balance between the values originally considered in KBM10. Meanwhile, the value of parameter $T_0$ controlling the influence of the dryness in the stochastic model is increased to yield more dramatic dependence on the moisture field.

The transition time scales (provided in Table 4) also play a critical role in the intermittency of the solution. A crucial design feature of the present parameter regime is the parity between congestus birth and decay time scales. While previous studies of organized tropical convection (e.g., Johnson et al. 1999; Mapes 2000, Khouider and Majda 2006, etc.)
suggest that cloud decay time scales are generally longer than cloud creation time scales, we choose to relax this proposition for congestus clouds. In this particular model, fast decay of the congestus clouds can be physically justified as a proxy for detrainment mechanism observed in nature (but otherwise missing in the model). While individually, neither of the above parameters leads to dramatic improvement in variability, all the parameter changes combined with closure modifications (formulated in the previous section) break the stochastic resonance observed in the KBM10 and yield the highly intermittent solution described above.

b. Sensitivity of single column simulations to SST variation

While additional sensitivity studies might further benefit our understanding of the performance of the stochastic multicloud model, one of the central results of this paper is the reproduction of a Walker type climatology and coherent variability, through the introduction of an SST gradient. Accordingly, the most crucial study of the performance of the model in such setting is the sensitivity of the parameterization to the SST variation. While, the Walker circulation is inherently a spatially correlated phenomenon, here we try to gain some insight from the single column simulation by artificially perturbing the SST around a fixed RCE state. Namely we introduce the spatial parameter $x$ and impose artificially

$$\theta_{eb}^*(x) = 5 \cos \left( \frac{4\pi x}{40000} \right) + 10K$$

(7)
within an interval of 20,000 km of a fictitious 40,000 km domain and \( \theta^*_{eb} = 5 \) K everywhere else as in Khouider and Majda (2007) and KM08a. Since the same perturbation profile will be used in the full \((x, t)\) simulations of the following section we choose to name this simulation a “mock” warm pool due to the artificial nature of the spatial dependency in the uncoupled column model.

The results are shown in Figure 6. The top two panels present the plots of the large scale variables \( \theta_1, \theta_2, \theta_{eb} \) and \( q \), while the next two plots show the cloud fractions and associated heating profiles as functions of the variable \( x \). Each point represents the 50 day average of the single column simulation with corresponding SST perturbation (plotted at the bottom panel of the figure). It is clear that an increase in ABL equivalent potential temperature in the “center” of this mock warm pool leads to increased congestus activity, which in turn leads to abundance of deep and consequently stratiform convection in the high SST region. The increase in convection leads to heating of the first and second baroclinic mode. Deep convective heating is more intermittent in low SST areas and on average lower but more persistent in high SST area. Meanwhile, the cloud fractions for all three cloud types peak at the center of the mock warm pool. The lack of the pronounced deep convective heating peak in the center points to the importance of the spatial effects, most importantly moisture convergence. While, the experiment does not produce a realistic model for the Walker climatology, the parameterization is not overly sensitive to SST variation (convection in various amounts exists for all perturbed states). In fact, this study is an analog of single column analysis for the homogeneous SST simulations and can be used effectively to benchmark the stochastic multi-cloud model in warm pool type simulations as illustrated above.
4. Stochastic multicloud model with spatial variation for moderate GCM resolution

We solve the equations described in Tables 1, 2 and 3 in \((x, t)\) variables on a 40 000 km periodic ring representing the perimeter of the earth at the equator. We use an operator time-splitting strategy where the conservative terms are discretized and solved by a nonoscillatory central scheme while the remaining convective forcing terms are handled by a second-order Runge–Kutta method (Khouider and Majda 2005a,b). The stochastic component of the scheme is resolved using Gillespie’s exact algorithm described in the previous section. The coupled model is integrated for a total period of 1250 days with a mesh size of 40 km and a time step of 2 minutes. The first 250 days are discarded as a transient period, while the last 1000 days are used for the calculation of the statistics. The last thirty days of the simulation are used to display the features of the solution.

a. Homogenous SST background: Aquaplanet

The simulation for a homogeneous SST background is presented in Figures 7 and 8. The first figure shows the velocity and temperature modes, while the second figure shows the moisture and ABL equivalent temperature followed by deep and congestus heating, as well as deep and congestus cloud fractions. Considering the contours of deep and congestus heating we note some of the prominent features of the simulation: synoptic and mesoscale convective systems, large intermittent congestus cloud decks, convectively coupled waves and small scale convective events. As was the case with the single column results, the boundary
layer temperature is the main source of the variability. The top right panel of Figure 8 shows the ABL equivalent potential temperature which correlates with the large peaks of congestus heating. These large congestus cloud decks moisten the lower troposphere and create favorable environment for large deep convective peaks. Radiating away from the near stationary decks are convectively coupled waves which move with an average speed of 17 m s$^{-1}$. Smaller scale structures such as intermittent congestus and deep convective events can be seen throughout the domain consistent with observations.

The congestus cloud decks appear intermittently in space and in time, and remain stationary while generating gravity and convectively coupled waves. While the average lifespan of the convectively coupled waves seems to be on the order of a week, the waves travel far enough to interact with convectively coupled waves from other cloud decks. As is the case with the single column results, the homogeneous SST simulations show a variety of the convective events of different scales and magnitudes. While large congestus cloud decks give birth to large precipitation events and the convectively coupled waves, the convectively coupled waves and the deep convective events themselves carry smaller amounts of congestus cloud cover. The structure of the waves is considered in the next subsection.

b. Warm pool

Now we impose an SST gradient for the background climate so that $\theta_{eb}^*$ is raised above its spatial mean by up to 5 K over a warm pool region of about 10 000 km in extent and lowered by the same amount outside the warm pool, as in Khouider and Majda (2007) (see Eq 3).
First we show the zonal structure of the resulting mean circulation in Figure 9. The simulation results in a nontrivial Walker like circulation with 10 m/s maximum horizontal velocity and 2 cm/s maximum updraft. The high SST creates an enhanced convection region inside of the warm pool resulting in a peak of deep convection in the center of the warm pool that drives the circulation, meanwhile on the edges of the warm pool the pronounced peaks in deep convection are due to intermittent convectively coupled waves radiating away from the warm pool (going through a mature stage then losing strength and moisture as they move away from the warm pool). It is important to recall that the “mock” warm pool simulation, that tracks the response of single column to SST variation, lacks the three peak deep convective maximum structure. We claim that enhanced deep convective heating inside the warm pool is due to correlated spatial effects, such as moisture convergence, as is the case for peaks on the boundary of the warm pool associated with transient convectively coupled waves. Small scale deep convective heating exists in the low SST zone; it is fueled by low first baroclinic potential temperature anomalies (high relative humidity). It is worth mentioning that the stochastic model avoids the sharp peaks characteristic of the deterministic model in the suboptimal regime; the stochastic model allows non trivial interactions between the waves and the warm pool forcing. The vertical structure of the resulting simulation can rival the ones produced by high resolution CRMs. For example Grabowski et al. (2000) reports circulation with a somewhat weaker horizontal velocity (8 m/s) and a stronger updraft (4 cm/s) as results of an SST gradient simulation in CRM runs on smaller 4000 km domain with similar vertical structure. The variability in the present stochastic simulation qualitatively resemble that in these vastly more expensive CRM simulations.

The structure of the deviations from the mean circulation, i.e, the waves, is plotted in
Figures 10 and 11 following the same format as Figures 7 and 8. The main driver of the circulation is the presence of large congestus cloud decks and associated large scale deep precipitation events. While in the homogeneous background simulation these cloud decks are intermittently created throughout the domain, the imposed SST gradient leads to the localization of the large congestus decks inside the high SST region. As in the homogenous background scenario, convectively coupled waves radiate away from the large congestus cloud decks that are now confined to the warm pool. These structures are short lived but create drier waves that propagate further away. The resulting precipitation profiles again resemble CRM simulations of Grabowski et al. (2000), where convectively coupled wave organization is observed in the high SST area. However, unlike Grabowski et al. (2000), intermittent deep convection episodes are also observed away from the warm pool region, perhaps, due to our much larger spatial domain.

The most interesting physical structures in the simulation are the CCW that move away from the warm pool at 17 m/s. The structure of these waves is illustrated in Figure 12, the waves are averaged in the moving reference frame resulting in maximum horizontal velocity of 3 m/s and 3 cm/s updraft. As was the case in the single column simulation, both cloud fractions and heating field exhibit congestus to deep to stratiform pattern consistent with the design principles of the multicloud model. Furthermore, the snapshots of the evolution of the wave reveal that the initial stage of the wave is dominated by congestus clouds near the warm pool. As the wave moves away from the warm pool area, deep convection peaks around 5 000 km away from the center of the warm pool. The \((x, z)\) structure of the heating field reveals characteristic tilt observed in nature (Wheeler and Kiladis, 1999; Kiladis et al. 2005) and consistent with the progressive deepening of convection from shallow to congestus
to deep to stratiform seen in detailed small domain CRM simulations (Waite and Khouider 2009).

Compared to the reference deterministic parameterization, the stochastic model avoids the oversensitivity to SST variation and reproduces a realistic Walker circulation (without the extremely sharp peak in deep convection), which is qualitatively similar to the high computational cost CRM results of Grabowski et al. (2000). The structure of the convectively coupled waves is greatly improved as well as the interactions between the mean circulations and mesoscale convective systems.

5. **Stochastic multicloud model with coarse GCM resolution and scaling**

This section considers scalability of the stochastic multicloud parameterization to the coarse GCM resolution (160 km). We propose an intuitive time scale dilation, which preserves the variability and statistical structure of coherent features described in the medium resolution study of the previous section. The qualitative analysis of the variability of the model is done in the warm pool setting, through judicious qualitative comparison of the scaled stochastic model to the deterministic GCM parameterization with clear deficiencies.

The model for equatorial flow can be considered as a chain of rectangular lattices. All the results described thus far have been achieved by considering rectangular lattices of size 40 km each occupied by $n \times n$ sites where $n$ is set to 30. In this framework, an individual convective element has an area of $1.3^2 \text{ km}^2$ and corresponds to a transition time scale on the
order of hours (as reported in Table 4). When the model is applied to a coarser resolution of 160 km, it is natural to consider modifications to the number of convective elements and their corresponding transition time scales. Naturally, the number of convective elements can be increased to $n=120$, preserving the area to time scale ratio for all convective elements. This however is a step towards the mean field limit (KBM10) and naturally leads to almost deterministic dynamics with reduced intermittency. Alternatively, we may consider the role of time scale adjustment parameter $\tau_{\text{grid}}$ that is set to one in the previous medium resolution simulation. Keeping the number of elements the same, we consider making transition time scales longer by increasing in the value of $\tau_{\text{grid}}$. The equilibrium distribution of convective elements is invariant under this transformation. Thus, keeping total number of convective elements the same, we simply associate larger convective sites with larger time scales, without disturbing the equilibrium structure of the solution. While it is not possible to use this argument to extend the model fully to a very coarse synoptic scale grid, both 40 km and 160 km resolutions are well within the mesoscale framework of the proposed stretched building block hypothesis (Mapes et al. 2006). Alternatively, variation of the parameter $\tau_{\text{grid}}$ can be viewed as a systematic sensitivity study of the effects of the time scales on the coarse resolution stochastic parameterization.

Motivated by the above discussion, we consider a set of numerical simulations in the warm pool environment. The strength of the mean circulation and standard deviation of deep convective and congestus heating fields are summarized in Table 5. As expected, an increase in the number of cloud sites, corresponding to a shift towards the mean field equations, results in a decrease in the variability of the solutions. While the mean field behavior of parameterization is interesting, the decrease in variability of the solution is
counterproductive. Thus for the remainder of the section we fix the number of cloud sites at \( n = 30 \) and experiment with the time scale adjustment only. The values of time scale dilation parameter \( \tau_{grid} = 1, 3, 4, 5, 6 \) and 16 are considered. Notice that the case of \( \tau_{grid} = 1 \) corresponds to a simple increase in the resolution of the stochastic lattice without any parameter changes and leads to a decrease of variability. Generally, the variance of congestus heating goes down as larger time scale dilations are introduced, meanwhile the variance of deep convection peaks near \( \tau_{grid} = 5 \). While the intuition suggests a spatial scaling of 16 (which preserves the area to time scale ratio), the simulations in this regime are characterized by a weak variability. Nonetheless, even this weakly fluctuating circulation of the stochastic model produces a variability comparable to the deterministic parameterization simulations on the coarse GCM grid, with greatly improved structure of mean circulation and convectively coupled waves.

Overall, the solution with \( \tau_{grid} = 4 \) has a balance of deep and congestus convection that produces the highest updraft of the simulation set. The mean of this circulation is plotted in Figure 13. The heating structure, which resembles the moderate resolution simulation, is characterized by the trimodality of the convective heating field. As before, the deep convective heating peaks outside of the warm pool correspond to the mature stage of CCWs moving away from the large and intermittent congestus cloud decks found in the center of the warm pool. The main difference between the moderate and coarse resolution simulations is that the deep convective peak in the center is much higher in the coarse resolution. The increase in the maximum updraft of the coarse resolution simulation correlates to the enhanced deep convective heating inside of the warm pool. As for the moderate grid counterpart, the coarse grid stochastic parameterization avoids the non-physical sharp peak seen in the determinis-
tic parameterization. The deviations from the mean are plotted in Figures 14, 15. We note that while the complexity of the structures is somewhat reduced, the mean features of the moderate resolution simulations are persevered. As before, large congestus cloud decks are confined to the warm pool area triggering convectively coupled waves that move away from the warm pool. The structure of such waves, given Figure 16, qualitatively resembles its moderate resolution counterpart (plotted in Fig.12). Small scale features associated with decaying CCW as well as random convective episodes are seen outside of the warm pool.

6. Concluding summary and discussion

Here a modified version of the stochastic multicloud model is used to study flows above the equator without rotation effects. As in KBM10, the model is based on a coarse grained Markov chain lattice model where each lattice site takes discrete values from 0 to 3 according to whether the site is clear sky or occupied by a congestus, deep or stratiform cloud. The convective elements of the model interact with each other and with the large scale environmental variables through CAPE and middle troposphere dryness. A few changes in the closure formulations are also introduced in order to improve the variability and the overall qualitative behavior of stochastic model. The major modifications to the original model include a direct dependence of congestus cloud cover on low level CAPE, a simplified stratiform heating closure and the inclusion of stratiform rain in the precipitation budget. Additionally, minor changes in tuning parameters and time scales are validated in single column simulations mimicking typical behavior of a GCM grid box. In this context, the combined effect of the changes yields highly intermittent solutions that capture the progres-
sive deepening of tropical convection on multiple scales. In particular, small scale convective events precondition the environment for large scale precipitation. While both small and large convective events follow congestus to deep to stratiform patterns, the prominent events are characterized by larger deep convective heating relative to the congestus component (due to direct clear sky to deep convection transitions in the preconditioned environment).

The parameterization is used in an aquaplanet setting to study flows above the equator without rotation effects on a moderate (40 km)–resolution domain. Detailed simulations show a menagerie of intermittent synoptic and mesoscale convective systems as well as the emergence of 17 m/s convectively coupled waves originating from large congestus cloud decks. Adhering to the design principles of the model, these intermittent and unstable waves have the characteristic congestus to deep to stratiform heating pattern and tilted heating field observed in nature. The introduction of an SST gradient leads to a localization of the large congestus cloud decks within the high SST region which in turn give rise to large deep convective features that drive a more realistic Walker type circulation. The convectively coupled waves propagate away from the boundaries of the congestus cloud decks resulting in high variability, which is further enhanced by intermittent small scale convective features away from the warm pool area. Both the structure of the mean circulation and waves are comparable to the results of CRM simulations of Grabowski et al. (2000). The advantages of using the stochastic parameterization are particularly apparent when the results are compared to the suboptimal deterministic (conventional-like) parameterization in this paradigm setting.

The design principle from KBM10 allows for a natural transition time scale dilation which improves the performance of the model on a coarse GCM size mesh (160 km).
study of the scaling parameter, in the SST gradient setting, shows that this principle can be successfully applied to produce coarse simulations that retain the variability and the statistical structure of coherent features of the medium resolution solution. For both the medium and coarse resolutions, the variability of the stochastic parameterization exceeds the variability of the deterministic parameterization run over a medium resolution grid. Furthermore, the variability comes from coherent structures as in nature. In the present idealized paradigm setting, the above results illustrate that the stochastic parameterization can successfully address the problem of missing tropical variability in deterministic GCM parameterizations.

It is worthwhile noting here that the main differences between the stochastic and deterministic parametrizations reside in 1) the fluctuations of the heating rates induced by the stochastic area fractions and 2) the systematic transitions between the various cloud types, essentially from congestus to deep and from deep to stratiform. All other parameters and closure formulations remained identical. In essence, we demonstrated here that this multicloud “stochasticization” framework can be easily exported to conventional cumulus parameterizations to improve the ability of operational GCMs to simulate the variability of organized tropical convection and convectively coupled waves. Such studies are currently being conducted by the authors in collaboration with other scientists and will be reported elsewhere in the near future.

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<th>Name</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum, $j$th mode, $j = 1, 2$</td>
<td>$\frac{\partial u_j}{\partial t} - \partial_x \theta_j = -Cdu_0u_j - \frac{1}{\tau_R}u_j$</td>
</tr>
<tr>
<td>Potential temperature, 1st mode</td>
<td>$\frac{\partial \theta_1}{\partial t} - \partial_x u_1 = H_d + \xi_s H_s + \xi_c H_c + S_1$</td>
</tr>
<tr>
<td>Potential temperature, 2nd mode</td>
<td>$\frac{\partial \theta_2}{\partial t} - \frac{1}{4} \partial_x u_1 = H_c - H_s + S_2$</td>
</tr>
<tr>
<td>Radiative cooling</td>
<td>$S_i = -Q^0_{R,i} - \tau_D^{-1}\theta_i$</td>
</tr>
<tr>
<td>Free tropospheric moisture</td>
<td>$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}[(u_1 + \tilde{\alpha}u_2)q + \tilde{Q}(u_1 + \tilde{\lambda}u_2)] = -P + \frac{D}{H_T}$</td>
</tr>
<tr>
<td>Boundary layer equivalent potential temperature</td>
<td>$\frac{\partial \theta_{eb}}{\partial t} = \frac{1}{h_b}(E - D)$</td>
</tr>
<tr>
<td>Downdrafts</td>
<td>$D = m_0[1 + \mu(H_s - H_c)/Q^0_{R,1}]^+ \Delta_m \theta_e$</td>
</tr>
<tr>
<td>Sea surface evaporation flux</td>
<td>$\frac{E}{h_b} = \tau_e^{-1}(\theta_{eb}^* - \theta_{eb})$</td>
</tr>
</tbody>
</table>
Table 2. Closure differences between the stochastic and the deterministic parameterizations.

<table>
<thead>
<tr>
<th>Name</th>
<th>Stochastic</th>
<th>Deterministic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Congestus heating closure</td>
<td>$H_c = \sigma_c \bar{\alpha} \sqrt{CAPE_i^+}$</td>
<td>-</td>
</tr>
<tr>
<td>Stratiform heating closure</td>
<td>$H_s = \alpha_s [\sigma_s \bar{Q} + \frac{1}{\tau_c(\sigma_s)} (a_1 \theta_{eb} + a_2 q - a_0 (\theta_1 + \gamma_2 \theta_2))]^+$</td>
<td>-</td>
</tr>
<tr>
<td>Stratiform heating closure</td>
<td>$H_s = \alpha_s [\sigma_s \bar{Q} + \frac{1}{\tau_c(\sigma_s)} (a_1 \theta_{eb} + a_2 q - a_0 (\theta_1 + \gamma_2 \theta_2))]^+$</td>
<td>-</td>
</tr>
<tr>
<td>Stratiform heating dynamics</td>
<td>-</td>
<td>$\frac{\partial H_s}{\partial t} = \frac{1}{\tau_s} (\alpha_s H_d - H_s)$</td>
</tr>
<tr>
<td>Congestus heating dynamics</td>
<td>-</td>
<td>$\frac{\partial H_c}{\partial t} = \frac{1}{\tau_c} (\alpha_c \lambda Q_c^+ - H_c)$</td>
</tr>
<tr>
<td>Deep convection dynamics</td>
<td>-</td>
<td>$H_d = (1 - \Lambda) Q_d^+$</td>
</tr>
<tr>
<td>Maximum energy available for deep convection</td>
<td>$CAPE = C\bar{APE} + R(\theta_{eb} - \gamma(\theta_1 + \gamma_2 \theta_2))$</td>
<td>$Q_d = \bar{Q} + \tau_{\text{conv}}^{-1} [a_1 \theta_{eb} + a_2 q - a_0 (\theta_1 + \gamma_2 \theta_2)]^+$</td>
</tr>
<tr>
<td>Maximum energy available for congestus convection</td>
<td>$CAPE_c = C\bar{APE} + R(\theta_{eb} - \gamma(\theta_1 + \gamma_2 \theta_2))$</td>
<td>$Q_c = \bar{Q} + \tau_{\text{conv}}^{-1} [\theta_{eb} - a_0'(\theta_1 + \gamma_2 \theta_2)]^+$</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
<td>Description</td>
</tr>
<tr>
<td>---------------</td>
<td>---------------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$h_b/H_T$</td>
<td>500 m /16 km</td>
<td>ABL depth/Free troposphere depth</td>
</tr>
<tr>
<td>$Q_{R1}$</td>
<td>1 K/day</td>
<td>First baroclinic radiative cooling rate</td>
</tr>
<tr>
<td>$Q_{R2}$</td>
<td>Determined at RCE</td>
<td>Second baroclinic radiative cooling rate</td>
</tr>
<tr>
<td>$\xi_s$</td>
<td>0.4/0.5</td>
<td>Stratiform contribution to first baroclinic mode for stochastic/deterministic model</td>
</tr>
<tr>
<td>$\bar{Q}$</td>
<td>0.9/ 6.5</td>
<td>Background moisture stratification</td>
</tr>
<tr>
<td>$\bar{\lambda}/\bar{\alpha}$</td>
<td>0.8/0.1</td>
<td>Coefficient of $u_2$ in linear / nonlinear moisture convergence</td>
</tr>
<tr>
<td>$m_0$</td>
<td>Determined at RCE</td>
<td>Large-scale background downdraft velocity</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.25</td>
<td>Contribution of convective downdrafts to $M_d$</td>
</tr>
<tr>
<td>$\alpha_s/\alpha_c$</td>
<td>0.25/ 0.1</td>
<td>Stratiform/Congestus adjustment coefficient</td>
</tr>
<tr>
<td>$\tau_R/\tau_D$</td>
<td>75 days / 50 days</td>
<td>Rayleigh drag /Newtonian cooling time scale</td>
</tr>
<tr>
<td>$\tau_s/\tau_c$</td>
<td>3 hours / 1 hour</td>
<td>Stratiform/Congestus adjustment time scale</td>
</tr>
<tr>
<td>$\tau_{conv}$</td>
<td>2 hours</td>
<td>Convective time scale</td>
</tr>
<tr>
<td>$\tau_e$</td>
<td>Determined by RCE</td>
<td>Surface evaporation time scale</td>
</tr>
<tr>
<td>$\tau_T$</td>
<td>8 hours</td>
<td>Momentum entrainment time scale</td>
</tr>
<tr>
<td>$\bar{Q}$</td>
<td>Determined at RCE</td>
<td>Bulk convective heating at RCE</td>
</tr>
<tr>
<td>$a_1/a_2$</td>
<td>0.45/0.55</td>
<td>Relative contribution of $\theta_{eb}$ / q to deep convection</td>
</tr>
<tr>
<td>$a_0/a'_0$</td>
<td>7 / 1.5</td>
<td>Dry convective buoyancy freq. in deep/congestus eq.</td>
</tr>
<tr>
<td>$\gamma_2/\gamma'_2$</td>
<td>0.1 / 2</td>
<td>Relative contribution of $\theta_2$ to deep /congestus heating</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.1</td>
<td>Relative contribution of $\theta_2$ to $\theta_{em}$</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.001</td>
<td>Surface drag coefficient</td>
</tr>
<tr>
<td>$u_0$</td>
<td>2 m/s</td>
<td>Strength of turbulent fluctuations</td>
</tr>
<tr>
<td>$CAPE_0$</td>
<td>200 J/Kg</td>
<td>reference values of CAPE</td>
</tr>
<tr>
<td>$T_0$</td>
<td>10 K</td>
<td>reference values of dryness</td>
</tr>
<tr>
<td>Transition</td>
<td>Transition Rate</td>
<td>Time scale (hours)</td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>---------------------------------------------------------------------------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>Formation of congest</td>
<td>$R_{01} = \frac{1}{\tau_{01}} \Gamma(C_l) \Gamma(D)$</td>
<td>$\tau_{01} = 1 \tau_{grid}$</td>
</tr>
<tr>
<td>Decay of congestus</td>
<td>$R_{10} = \frac{1}{\tau_{10}} \Gamma(D)$</td>
<td>$\tau_{10} = 1 \tau_{grid}$</td>
</tr>
<tr>
<td>Conversion of congest to deep</td>
<td>$R_{12} = \frac{1}{\tau_{12}} \Gamma(C)(1 - \Gamma(D))$</td>
<td>$\tau_{12} = 3 \tau_{grid}$</td>
</tr>
<tr>
<td>Formation of deep</td>
<td>$R_{02} = \frac{1}{\tau_{02}} \Gamma(C)(1 - \Gamma(D))$</td>
<td>$\tau_{02} = 3 \tau_{grid}$</td>
</tr>
<tr>
<td>Conversion of deep to stratiform</td>
<td>$R_{23} = \frac{1}{\tau_{23}}$</td>
<td>$\tau_{23} = 3 \tau_{grid}$</td>
</tr>
<tr>
<td>Decay of deep</td>
<td>$R_{20} = \frac{1}{\tau_{20}} (1 - \Gamma(C))$</td>
<td>$\tau_{20} = 3 \tau_{grid}$</td>
</tr>
<tr>
<td>Decay of stratiform</td>
<td>$R_{30} = \frac{1}{\tau_{30}}$</td>
<td>$\tau_{30} = 5 \tau_{grid}$</td>
</tr>
</tbody>
</table>
Table 5. Mean circulation strength and variability of heating fields for the stochastic and deterministic parameterizations under different scalings

<table>
<thead>
<tr>
<th>Model</th>
<th>grid</th>
<th>scaling</th>
<th>warm pool max (U, W)</th>
<th>std($H_d$)</th>
<th>std($H_c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic</td>
<td>40km</td>
<td>$\tau_{grid} = 1, n=30^2$</td>
<td>(10m/s, 2cm/s)</td>
<td>2.14 K/Day</td>
<td>2.83 K/Day</td>
</tr>
<tr>
<td>Stochastic</td>
<td>160km</td>
<td>$\tau_{grid} = 1,n=120^2$</td>
<td>(12m/s, 3cm/s)</td>
<td>1.34 K/Day</td>
<td>1.89 K/Day</td>
</tr>
<tr>
<td>Stochastic</td>
<td>160km</td>
<td>$\tau_{grid} = 1,n=30^2$</td>
<td>(12m/s, 3cm/s)</td>
<td>1.67 K/Day</td>
<td>2.41 K/Day</td>
</tr>
<tr>
<td>Stochastic</td>
<td>160km</td>
<td>$\tau_{grid} = 3,n=30^2$</td>
<td>(12m/s, 4cm/s)</td>
<td>1.80 K/Day</td>
<td>2.21 K/Day</td>
</tr>
<tr>
<td>Stochastic</td>
<td>160km</td>
<td>$\tau_{grid} = 4,n=30^2$</td>
<td>(12m/s, 6cm/s)</td>
<td>1.96 K/Day</td>
<td>2.07 K/Day</td>
</tr>
<tr>
<td>Stochastic</td>
<td>160km</td>
<td>$\tau_{grid} = 5,n=30^2$</td>
<td>(11m/s, 5cm/s)</td>
<td>2.09 K/Day</td>
<td>1.80 K/Day</td>
</tr>
<tr>
<td>Stochastic</td>
<td>160km</td>
<td>$\tau_{grid} = 6,n=30^2$</td>
<td>(11m/s, 5cm/s)</td>
<td>1.66 K/Day</td>
<td>1.35 K/Day</td>
</tr>
<tr>
<td>Stochastic</td>
<td>160km</td>
<td>$\tau_{grid} = 16,n=30^2$</td>
<td>(10m/s, 3cm/s)</td>
<td>0.49 K/Day</td>
<td>0.89 K/Day</td>
</tr>
<tr>
<td>Deterministic</td>
<td>40km</td>
<td>-</td>
<td>(4m/s, 4cm/s)</td>
<td>0.97 K/Day</td>
<td>0.14 K/Day</td>
</tr>
<tr>
<td>Deterministic</td>
<td>160km</td>
<td>-</td>
<td>(5m/s, 4cm/s)</td>
<td>0.55 K/Day</td>
<td>0.14 K/Day</td>
</tr>
</tbody>
</table>
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