

1 **Multi-scale interactions in an idealized**
2 **Walker cell: Simulations with sparse**
3 **space-time superparameterization**

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ABSTRACT

6 This paper discusses the Sparse Space and Time SuperParameterization (SSTSP) algorithm
7 and evaluates its ability to represent interactions between moist convection and the large-
8 scale circulation in the context of a Walker cell flow over a planetary scale two-dimensional
9 domain. The SSTSP represents convective motions in each column of the large-scale model
10 by embedding a cloud-resolving model, and relies on a sparse sampling in both space and
11 time to reduce computational cost of explicit simulation of convective processes. Simulations
12 are performed varying the spatial compression and/or temporal acceleration, and results are
13 compared to the cloud-resolving simulation reported previously. The algorithm is able to
14 reproduce broad range of circulation features for all temporal accelerations and spatial com-
15 pressions, but significant biases are identified. Precipitation tends to be too intense and too
16 localized over warm waters when compared to the cloud-resolving simulations. It is argued
17 that this is because coherent propagation of organized convective systems from one large-
18 scale model column to another is difficult when superparameterization is used, as noted in
19 previous studies. The Walker cell in all simulations exhibits low-frequency variability on
20 time scale of about 20 days, characterized by 4 distinctive stages: suppressed, intensifica-
21 tion, active, and weakening. The SSTSP algorithm captures spatial structure and temporal
22 evolution of the variability. This reinforces the confidence that SSTSP preserves fundamen-
23 tal interactions between convection and the large-scale flow, and offers a computationally
24 efficient alternative to traditional convective parameterizations.

1. Introduction

The interplay between moist convection and the large-scale flow is the fundamental feature of the tropical atmosphere. However, the extreme range of spatial and temporal scales involved makes it difficult to resolve all relevant processes in numerical models. In large-scale models, this issue has traditionally been addressed through the use of convective parameterizations that account for effects of convective motions on the mean atmospheric temperature and humidity profiles. It is well recognized, however, that convective parameterizations fail to reproduce many important features of the tropical atmosphere. This is partly because many aspects of convection, such as downdrafts, cold pools, and mesoscale organization, are either excluded or poorly represented in the parameterizations. Moreover, the parameterizations often do not reproduce the intrinsic intermittency of moist convection. This motivates the development of new approaches to improve the representation of convection in multi-scale simulations of the tropical atmosphere.

One way to improve such simulations is to take advantage of cloud-resolving modeling. Cloud models emerged in 1970s (e.g., Steiner 1973; Schlesinger 1975; Klemp and Wilhelmson 1978; Clark 1979) to study individual clouds in short simulations (tens of minutes) and typically applied idealized forcing techniques (e.g., initiating cloud development via a warm bubble). More recently such models have been used in significantly longer simulations (days and weeks) and applying large computational domains. Such simulations are often driven by observationally-based time-evolving larger-scale forcings and allow better comparisons with observations (e.g., Grabowski et al. 1996, 1998; Xue and Randall 1996; Xue et al. 2002, Fridlind et al. 2012, among many others). Cloud-resolving models solve non-hydrostatic governing equations and allow convective development in conditionally unstable conditions. The horizontal resolution of ~ 1 km is high enough for the simulation of dynamical evolution of individual clouds, with microphysical, turbulent and radiative processes needed to be parameterized. Explicit representation of cloud dynamics allows capturing key features that convective parameterizations struggle with. During the last 30 years, many studies

52 focused on statistical response of cloud ensembles to the large-scale forcing over a limited
53 area (e.g., Soong and Ogura 1980, Soong and Tao 1980, Tao and Soong 1986). So far,
54 cloud-resolving models appear superior to any kind of convective parameterization, as found
55 by comparing model results to observations. However, the computational cost still severely
56 limits global cloud-resolving simulations and alternative approaches need to be explored. An
57 important application of cloud-resolving modeling in the context of global simulation is vali-
58 dation and improvement of other approaches designed to estimate feedbacks from convective
59 to mesoscale, synoptic and global scales.

60 Rescaling approaches have also been suggested to extend cloud-resolving modeling to
61 global simulations. The underlying idea is to artificially reduce the scale separation between
62 convective and planetary scales, and thus to make explicit simulation of convection com-
63 putationally feasible in global domains. The Diabatic Acceleration and Rescaling (DARE)
64 approach (Kuang et al. 2005) and the hypo-hydrostatic approach (Pauluis et al. 2005, Garner
65 et al. 2005) are examples of such techniques. In DARE, the Earth’s diameter is reduced, the
66 rotation rate is increased, and diabatic processes are accelerated. In the hypo-hydrostatic
67 approach, the vertical acceleration is rescaled. Pauluis et al. (2005) have shown that both
68 approaches are mathematically equivalent and they reduce the scale separation between
69 convection and the planetary scale without affecting the dynamics at large scales. However,
70 changes in the behavior of convection due to the rescaling limit the applicability of these
71 methods. Nevertheless, they illustrate how mathematical rescaling can offer a computation-
72 ally efficient way to use cloud-resolving models in global simulations.

73 The second approach is to take advantage of a cloud-resolving model for global simula-
74 tions through the superparameterization methodology (Grabowski and Smolarkiewicz 1999;
75 Grabowski 2001, 2004; Randall et al. 2001). In this framework, a two-dimensional cloud-
76 resolving model with periodic lateral boundaries is embedded within each column of a global
77 model to simulate interactions between convective and global scales. The simulated large-
78 scale scale flow includes the convective feedback from small to large scales, and convective

79 scales respond to the forcing from the large-scale dynamics. In the original superparame-
80 terization (SP thereafter), convective feedback is calculated using a cloud-resolving model
81 applying horizontal domain equal (or approximately equal) to the large-scale model horizon-
82 tal gridlength. Grabowski (2001, section 3) simulated a 2D Walker cell of 4000 km horizontal
83 extent applying SP approach with different horizontal gridlengths (from 20 to 500 km) and
84 thus different extents of the SP model horizontal domain. Results from these simulations
85 were compared to the fully-resolved simulations (described in Grabowski et al. 2000) as well
86 as between each other. SP seemed reasonably successful in reproducing large-scale conditions
87 as simulated by the cloud-resolving model (e.g., dry subsidence and humid ascent regions,
88 large-scale flow featuring the first and second baroclinic modes, etc.). However, mesoscale
89 organization of convection and the strength of the quasi-two-day oscillations, the prominent
90 feature of the fully-resolved simulations, were significantly different between SP simulations.

91 Over the last ten years, SP has been tested in many studies of tropical dynamics.
92 Khairoutdinov et al. (2005) and DeMott et al. (2007) found that while Madden Julian
93 Oscillation (MJO) is missing from the standard Community Atmosphere Model (CAM),
94 it is simulated reasonably well with SP-CAM (i.e., the superparameterized CAM). They
95 report several important improvements in simulating tropical climatology, such as a more
96 realistic distribution of cirrus cloudiness or intense precipitation. However, some impor-
97 tant biases persist, for instance, too heavy precipitation over the western tropical Pacific
98 associated with the Indian monsoon or too low shallow-convection cloud fraction and light
99 rain across parts of the tropics and subtropics. Studies attempting to explain the reason
100 of excessive precipitation in western Pacific during monsoon periods typically find signifi-
101 cant correlation between moisture content in the column and precipitation. Thayer-Calder
102 and Randall (2009) suggest that the difference comes from contrasting profiles of convective
103 heating that excite different large-scale circulation (and thus affect surface wind and evap-
104 orative feedback) and subsequently differently moisten the troposphere. Luo and Stephens
105 (2006) argue that convection-evaporation feedback is the main culprit of excessive rain and

106 suggest that this may be due to the periodicity of SP’s cloud-resolving models leading to the
107 prolonged presence of precipitating convection at a given location.

108 The mathematical aspects of the SP implementation are important as illustrated by the
109 above examples and other studies (e.g., Grabowski 2004). Over the years, several algorithms
110 have been proposed to implement the SP framework. Here, we evaluate the ability of the
111 Sparse Space and Time SuperParameterization (SSTSP) to accurately reproduce the inter-
112 actions between convection and the large-scale flow. In the SSTSP framework, described
113 in more detail in the next section, the embedded cloud-resolving model applies horizontal
114 domain that is small in comparison to the horizontal gridlength of the large-scale model,
115 and for the time period that is short when compared to the time step of the large-scale
116 model. SSTSP combines the spatial compression used in the previous SP implementation
117 (Xing et al. 2009) with a temporal acceleration similar to the DARE and hypo-hydrostatic
118 rescaling, thus significantly increasing computational efficiency of the approach. As with the
119 original SP, the goal of SSTSP algorithm is to obtain statistically correct representation of
120 the convective impact on the large-scale flow at reduced computational cost.

121 Preliminary SSTSP testing reported in Xing et al. (2009) applied two-dimensional sim-
122 ulations of an idealized squall line propagating in a periodic horizontal domain of 1024 km.
123 Performance of SSTSP algorithm was examined for a range of environmental conditions that
124 differed in the prescribed vertical shear of the large-scale horizontal wind. SSTSP algorithm
125 seemed to capture propagation of the squall line and its speed. In particular, propagation
126 speed appeared to be strongly controlled by vertical profile of the large-scale shear, with no
127 significant drawbacks of the SSTSP algorithm. Contrasting convective organizations were
128 simulated for different shears, from squall line to decaying convection. This provided hope
129 for the SSTSP algorithm in simulations of different convective regimes for various large-scale
130 conditions. Furthermore, structural agreement was found for large-scale features of simu-
131 lated convective systems since pattern correlation was high for horizontal velocity or specific
132 humidity. However, the impact of SSTSP algorithm on large-scale features (e.g., the mean

133 temperature and moisture profiles) was severely limited because of the short simulation time
134 (36 hours) and relatively small computational domain.

135 Here, we investigate the accuracy of the SSTSP algorithm in reproducing interactions
136 between convection and the large-scale flow in an idealized Walker cell circulation. We
137 compare SSTSP results against the benchmark solution obtained with the cloud-resolving
138 model. The latter is described in more detail in Slawinska et al. (2014; SPMG hereinafter)
139 focusing on the intra-seasonal variability of the Walker cell with the time-scale of about 20
140 days. The low-frequency oscillation features four phases: the suppressed, intensification,
141 active, and weakening. Intensification of the circulation is associated with the broadening
142 of the large-scale ascent region, which in turn is strongly coupled to propagating synoptic-
143 scale systems. Details of the SSTSP framework and its implementation are given in section
144 2. Results of simulations applying the SSTSP framework are discussed in section 3 and
145 compared to the results from SPMG’s cloud-resolving model. Section 4 provides a discussion
146 of model results and concludes the paper.

147 **2. Model and experimental setup**

148 In this study, we use the anelastic nonhydrostatic atmospheric model EULAG (Smol-
149 larkiewicz and Margolin 1997; see Prusa et al. 2008 for a comprehensive review) applying
150 the SP methodology (Grabowski 2001; 2004) and implement the SSTSP framework as de-
151 scribed briefly below (see also Xing et al. 2009).

153 1) LARGE-SCALE AND CLOUD-RESOLVING MODEL EQUATIONS

The large-scale and cloud-resolving models calculate evolution of the large-scale Φ and small-scale φ variables:

$$\Phi = [U, W, \Theta, Q_v, Q_c, Q_p], \quad (1)$$

$$\varphi = [u, w, \theta, q_v, q_c, q_p]. \quad (2)$$

The variables are the horizontal (U and u) and vertical (W and w) velocities, potential temperature (Θ and θ), water vapor (Q_v and q_v), condensed water/ice (Q_c and q_c), and precipitating water/ice (Q_p and q_p) mixing ratios, the latter two following representation of moist thermodynamics of Grabowski (1998). Evolution of Φ and φ can be symbolically written as:

$$\frac{\partial \Phi}{\partial t} + A_\Phi = S_\Phi + F_\Phi^{CS} \quad (3)$$

$$\frac{\partial \varphi}{\partial t} + A_\varphi = S_\varphi + F_\varphi^{LS} \quad (4)$$

154 where $A_\Phi \equiv \frac{1}{\rho_o} \frac{\partial}{\partial X_j} (\rho_o U_j \Phi)$ and $A_\varphi \equiv \frac{1}{\rho_o} \frac{\partial}{\partial x_j} (\rho_o u_j \varphi)$ represent the large-scale and small-scale
 155 advection terms, respectively; S_Φ and S_φ represent various source terms in the large-scale
 156 and small-scale models (such as the buoyancy, pressure gradient, radiative cooling, surface
 157 fluxes, phase changes of the water substance, precipitation formation and fallout, gravity
 158 wave absorber, etc.); F_Φ^{CS} is the small-scale feedback; and F_φ^{LS} stands for the large-scale
 159 forcing. The latter two terms represent the coupling between the two models. The source
 160 terms S_Φ and S_φ need to be appropriately designed between the two models. For instance,
 161 the pressure gradient terms are independently formulated between the models (e.g., via the
 162 anelastic continuity equation). The horizontally-averaged vertical velocity at each level of
 163 the small-scale model has to vanish because of the periodic lateral boundary conditions,
 164 and the vertical velocity field cannot be coupled between the two models. Surface fluxes,
 165 radiative transfer, phase changes and precipitation formation/fallout are typically considered

166 in the small-scale model only and they affect the large-scale fields through the small-scale
 167 feedback. In general, one needs to ensure that a given source is included only once between
 168 the two models, that is, no double-counting takes place.

169 2) COUPLING PROCEDURE IN SP

The original implementation of the SP is as follows (cf. Grabowski 2004). Every large-scale grid, $\Delta X \times \Delta Z$, contains a cloud-resolving (small-scale) model that has $N_x \times N_z$ grid points of grid size $\Delta x \times \Delta z$, for which

$$\Delta X = N_x \Delta x; \Delta Z = \Delta z, \quad (5)$$

that is, the horizontal extent of the small-scale model domain is equal to the horizontal gridlength of the large-scale model, and the two models share the same vertical grid. For a given large-scale time step, ΔT , the evolution from time T to $T + \Delta T$ of the large-scale variable is calculated first:

$$\Phi|^{T+\Delta T} = \Phi|^{T} + \Delta T(A_\Phi + S_\Phi)|^{T+\Delta T} + \Delta T F_\Phi^{CS}|^{T}, \quad (6)$$

170 with $\Delta T(A_\Phi + S_\Phi)|^{T+\Delta T}$ standing for the transport and large-scale sources over the period
 171 ($T : T + \Delta T$) and $F_\Phi^{CS}|^{T}$ representing the small-scale feedback calculated at previous time,
 172 T , as given by Equation (10).

With the large-scale variables already known at $(T + \Delta T)$ the vertical profiles of large-scale forcing for the small-scale variables, $F_\varphi^{LS}|^T$, are formulated as follows:

$$F_\varphi^{LS}|^T = \frac{\Phi|^{T+\Delta T} - \langle \varphi|^{T} \rangle|_1^{N_x}}{\Delta T}, \quad (7)$$

where $\langle . \rangle|_1^{N_x}$ stands for the horizontal averaging over the N_x points of the small-scale model. With the large-scale forcing formulated as above and assumed constant for the large-scale time step, the small-scale model equations are advanced from T to $T + \Delta T$:

$$\varphi|^{T+\Delta T} = \varphi|^{T} + \sum_{i=1}^{N_t} \Delta t (A_\varphi + S_\varphi)|_{T+(i-1)\Delta t}^{T+i\Delta t} + \sum_{i=1}^{N_t} \Delta t F_\varphi^{LS}|^T, \quad (8)$$

over N_t time steps for which:

$$\Delta T = N_t \Delta t. \quad (9)$$

Finally, at the end of the large-scale model time step, average profiles of the small-scale feedback are formulated as:

$$F_{\Phi}^{CS}|^{T+\Delta T} = \frac{\langle \varphi|^{T+\Delta T} \rangle |1^{N_x} - \Phi|^{T+\Delta T}}{\Delta T}. \quad (10)$$

173 Repeating (8), (9), (10), and (12) allows stepping forward in time of the combined small-scale
174 and large-scale system.

175 3) SPARSE SPACE-TIME ALGORITHM

The sparse space-time algorithm (Xing et al. 2009) reduces the computational cost of the SP by decreasing the horizontal extent of small-scale domain by a factor of p_x (i.e., p_x smaller number of model columns; reduced space strategy) and the number of small-scale time steps by a factor of p_t (reduced time strategy). In such a case, the number of small-scale time steps in every large-scale time step and the number of columns in every large-scale grid, N_{p_t} and N_{p_x} , are given by:

$$N_{p_t} = \frac{N_t}{p_t}, \quad (11)$$

$$N_{p_x} = \frac{N_x}{p_x}. \quad (12)$$

As in the original SP, the evolution of the large-scale variables is calculated first according to (6). Then, profiles of the large-scale forcings are calculated for the small-scale domain of N_{p_x} horizontal columns similarly to (9):

$$F_{\varphi}^{LS}|^T = p_t \frac{\Phi|^{T+\Delta T} - \langle \varphi|^T \rangle |1^{N_{p_x}}}{\Delta T}, \quad (13)$$

but including p_t (i.e., adding the time rescaling of the large-scale forcing), and applying horizontal averaging over the rescaled small-scale domain (marked $\langle . \rangle |1^{N_{p_x}}$). Subsequently, accelerated evolution of the small-scale variables over N_{p_t} time steps are calculated similarly

to (10):

$$\varphi|^{T+\frac{\Delta T}{p_t}} = \varphi|^{T} + \sum_{i=1}^{N_{p_t}} \Delta t (A_\varphi + S_\varphi)|_{T+(i-1)\Delta t}^{T+i\Delta t} + \sum_{i=1}^{N_{p_t}} \Delta t F_\varphi^{LS}|^T. \quad (14)$$

The small-scale variables at the end of the large-scale time step, $\varphi|^{T+\Delta T}$, are assumed equal to the solution of (17) with accelerated forcing, that is,:

$$\varphi|^{T+\Delta T} = \varphi|^{T+\frac{\Delta T}{p_t}}. \quad (15)$$

Finally, profiles of the small-scale feedback are computed as:

$$F_\varphi^{CS}|^{T+\Delta T} = \frac{\varphi|^{T+\frac{\Delta T}{p_t}} - \Phi|^{T+\Delta T}}{\Delta T}. \quad (16)$$

176 Elementary considerations (similar to those involving Eqs. 10 and 11 in Grabowski 2004)
 177 document that the SSTSP algorithm outlined above ensures appropriate transfer of infor-
 178 mation between the small-scale and large-scale models despite spatial compression and tem-
 179 poral acceleration. For instance, if either S_Φ in (3) or S_φ in (4) is assumed constant, then
 180 the tendency due to this source is correctly passed from one model to another (i.e., from the
 181 large-scale to small-scale model for S_Φ and vice-versa for S_φ) when spatial compression and
 182 temporal acceleration are applied.

183 Beyond mathematical consistency, one should be also aware of physical limitations of the
 184 SSTSP methodology. For the spatial compression, small horizontal domain of the small-scale
 185 model may affect not only the statistical sampling of small-scale features, but their evolution
 186 as well, evolution of convective cells in particular. Since the mean vertical velocity within
 187 SP models at any level has to vanish (because of periodic lateral boundary conditions),
 188 the upward convective mass flux has to be balanced by the environmental subsidence. The
 189 key point is that the vertical development of convective clouds may be affected when the
 190 computational domain is reduced to a small number columns. The temporal acceleration is
 191 perhaps more difficult to interpret. The approach taken in Xing et al. (2009) and followed
 192 here (cf. Eq. 16) implies that the large-scale forcing is increased in proportion to the temporal
 193 acceleration factor p_t . The idea is that the original large-scale forcing has to be increased so

194 the small-scale processes can appropriately respond over the p_t -shorter time. An alternative
195 approach might be to keep the large-scale forcing unchanged, but instead increase the small-
196 scale feedback by p_t . In other words, the small-scale response to the original feedback would
197 be calculated only for the p_t -fraction of ΔT and linearly extrapolated (i.e., increased by a
198 factor of p_t) before applied to the large-scale model. Such a procedure would lead to the
199 same evolution in time of Ψ and φ for the case of a constant source. However, considering
200 fundamental differences between time scales involved in small-scale and large-scale processes,
201 extrapolation of the small-scale response seems more problematic than scaling up the large-
202 scale forcing.

203 *b. Experimental design*

204 Developments presented in the previous section are tested applying the Walker cell cir-
205 culation in the two-dimensional domain following SPMG. As in SPMG, the environmental
206 profiles come from a simulation of radiative-convective equilibrium applying a cloud-resolving
207 model SAM with NCAR CAM3 interactive radiation scheme (Khairoutdinov and Randall
208 2003). The planetary-scale circulation is driven by the surface fluxes and radiative cooling.
209 The sea surface temperature (SST) distribution is given by a cosine squared function, with
210 303.15 K in the center and 299.15 K at the periodic lateral boundaries. Radiative cooling
211 is given by the average profile of radiative tendency in the radiative-convective simulation
212 and by the relaxation term towards the equilibrium value of potential temperature with the
213 20-day timescale. More detailed description of the modeling setup can be found in SPMG
214 which discusses results from the cloud-resolving simulation that provide the reference for SP
215 simulations.

216 In SP simulations, the large-scale domain spans 40,000 km with horizontal and verti-
217 cal gridlengths of 48 km and 500 m, respectively. The large-scale time step is 180 s. The
218 cloud-resolving domain has horizontal and vertical gridlengths of 2 km and 500 m, respec-
219 tively, and the small-scale time step of 15 s. The simulations are run for 340 days, and

220 the last 290 days are analyzed. Because of the simulation length, no other SP setups (i.e.,
221 either larger or smaller large-scale model gridlength, cf. section 3 of Grabowski 2001) were
222 considered. Simulations with various time accelerations and space compressions are com-
223 pared. Horizontal domain of the cloud-resolving model is equal to the large-scale horizontal
224 gridlength (i.e., $p_x = 1$) or is reduced by a factor of 2 ($p_x = 2$) or 3 ($p_x = 3$). Also, for
225 every large-scale time step, time integration in cloud-resolving domains is performed for the
226 period either equal ($p_t = 1$) or two ($p_t = 2$), three ($p_t = 3$) and four ($p_t = 4$) times shorter
227 than the large-scale model time step. A simulation with a given spatial compression (p_x)
228 and temporal acceleration (p_t) will be referred to as "SSTSP $p_x p_t$ simulation". For instance,
229 a simulation with $p_x = 2$ and $p_t = 3$ will be called "SSTSP23 simulation". In total, 12
230 simulations are performed with different time accelerations and space compressions. We will
231 refer to them as "SSTSP simulations". SSTSP simulations are compared to the benchmark
232 case obtained with the cloud-resolving model and analyzed in SPMG, and referred to as the
233 "CRM simulation" thereafter.

234 **3. Results**

235 SSTSP simulations reproduce the key characteristics of the CRM simulation. In par-
236 ticular, large-scale overturning circulation is simulated in the large-scale domain, with the
237 large-scale ascent over warm pool and subsidence over cold SSTs. Similarly to the CRM
238 simulation, variability across wide range of scales is simulated. We start with a discussion
239 of the mean state. Subsequently, we present analysis of high- and low-frequency variability,
240 with the latter analyzed in more detail. The emphasis is on comparing the SP and CRM
241 simulations (the latter one documented in details in Slawinska et. al. (2014)) and evaluating
242 the impact of the spatial and temporal scaling factors p_x and p_t .

243 *a. The mean Walker cell circulation*

244 Figure 1 shows the time-averaged horizontal velocity field for the CRM and SSTSP sim-
245 ulations (the former already shown in Fig. 2a in SPMG) as well as the difference between
246 them. The CRM large-scale circulation features surface and mid-tropospheric mean hori-
247 zontal flows towards the highest SST in the center of the domain. The horizontal velocity
248 maxima are around 10 and 5 m s^{-1} at the surface and around 6-km altitude, respectively.
249 The upper-tropospheric outflow from the center of the domain features maximum velocities
250 of over 20 m s^{-1} . SSTSP simulations exhibit similar large-scale circulations, with low- and
251 mid-level convergence accompanied by the upper-tropospheric divergence over warm SSTs,
252 that is, with the first and second baroclinic modes. The most apparent difference between
253 CRM and SSTSP simulations is the narrower ascending region in the center of the domain
254 in SSTSP cases. Although the patterns and amplitudes of the horizontal flow are similar in
255 all simulations, the difference plots between CRM and SSTSP show significant deviations
256 that seem to increase with the spatial compression and temporal acceleration, with the SP
257 simulation without compression and acceleration (i.e., SSTSP11) being the closest to CRM
258 as one might expect. Although not shown in the figure, the differences depend primarily
259 on the horizontal extent of the SP domains (i.e., they increase with the increase of p_x), and
260 there seems to be no systematic impact of the temporal acceleration (i.e., increasing the p_t
261 parameter).

262 Figure 2 and 3 document the impact of spatial compression and time acceleration on the
263 mean (i.e., horizontal- and time-averaged) profiles of the potential temperature and water
264 vapor mixing ratio. Figure 2 shows the difference between profiles from SSTSP with various
265 spatial compressions and CRM. Mean profiles for the SSTSP11 simulation are close to CRM,
266 and the differences increase with the spatial compression. The SSTSP31 simulation features
267 up 8 K colder upper troposphere and up to 2 g/kg lower moisture in the lower troposphere
268 when compared to CRM. The relative humidity profiles (not shown) agree relatively well
269 below 8 km for all simulations and differ significantly above 10 km, with no obvious sensitivity

270 to the spatial compression. The differences between the temperature profiles are consistent
271 with a heuristic argument that reducing the horizontal extent of SP computational domains
272 (i.e., increasing p_x) makes convective overturning more difficult and leads to a colder upper
273 troposphere. The water vapor difference profiles can be explained by a narrower ascending
274 region in the center of the domain as illustrated in Fig. 1 and further quantified below. As
275 shown in Fig. 3, time acceleration leads to the mean temperature/moisture profiles that are
276 warmer/more humid, but the effects are significantly smaller than for the spatial compression,
277 especially for the moisture.

278 Figure 4 and 5 show spatial distributions of the difference between the SSTSP and CRM
279 simulations for the mean temperature and water vapor mixing ratio, respectively. The
280 differences are averaged over days 50 to 340. For the temperature, the patterns are dominated
281 by the differences in the mean profiles (cf. Fig. 2), with small gradients between regions with
282 high and low SST (i.e., mean ascent and mean subsidence). In the CRM simulation, the
283 temperature field at a given level is homogenized by convectively-generated gravity waves
284 that maintain small horizontal temperature gradient. Such a mechanism is also efficient in
285 SP simulations, including SSTSP, as documented by relatively small horizontal temperature
286 gradients in Fig. 4. Water vapor field, on the other hand, can only be homogenized by the
287 physical advection and the differences between SSTSP and CRM simulations are larger, as
288 shown in Fig. 5. The largest differences (in the absolute sense) are near the center of the
289 domain, likely because of the different width of the central ascending region and differences
290 in the large-scale circulation (cf. Fig. 1). The differences increase with the increase of the
291 spatial compression and temporal acceleration. The lower troposphere above 1 km is drier
292 in SSTSP than in the CRM, and in both the ascent and subsidence regions, perhaps with
293 the exception of the narrow zone over the coldest SSTs. The level of maximum difference
294 outside the central region at heights between 2 and 3 km corresponds to the low level cloud
295 tops (see below). Upper troposphere is drier at the warm pool edges, likely because of the
296 narrower region of deep convection in the SSTSP simulations.

297 Figure 6 and 7 show time-averaged mean fields and profiles of the cloud condensate mixing
 298 ratio, respectively. Fig. 6 shows that shallow convection occurs over the entire domain, while
 299 deep convection is confined to the warm pool. The region with deep convection narrows
 300 when the spatial compression and temporal acceleration increase. There are also systematic
 301 changes of the mean cloud condensate profiles as documented in Fig. 7. The figure documents
 302 the classical trimodal characteristics of the tropical moist convection: shallow, congestus,
 303 and deep (cf. Johnson et al. 1999), with the lower-tropospheric maximum associated with
 304 shallow convective clouds, and middle- and upper-tropospheric maxima marking detrainment
 305 levels from congestus and deep convection, respectively. Temporal acceleration results in
 306 a significant shift of the profiles towards higher values (factor of approximately 2 between
 307 panels *a* and *d*). Spatial compression for a given temporal acceleration has relatively smaller
 308 effect, with systematic decrease of cloud condensate above 5 km.

309 Figure 8 shows mass flux profiles for CRM and SSTSP simulations with various spa-
 310 tial compressions and temporal accelerations. Since these profiles are derived by averaging
 311 the cloud-model data, they represent the impact of the SSTSP methodology on convective
 312 transport. The SP simulation with neither spatial compression nor temporal acceleration
 313 (i.e., SSTSP11) gives the mean mass flux close to the one from the CRM simulation. Spatial
 314 compression (i.e., SSTSP31) leads to significantly reduced mass flux, arguably because of
 315 the impact of a reduced extent of the cloud-model computational domain on the convec-
 316 tive transport as argued at the end of section 2a. In contrast, temporal acceleration (i.e.,
 317 SSTSP13) leads to a significant increase of the mass flux, arguably because of the increase
 318 of the large-scale forcing (cf. Eqs. 7 and 13). Combining spatial compression and temporal
 319 acceleration (i.e., SSTSP33) results in the convective mass flux in between simulations with
 320 either spatial compression or temporal acceleration.

321 The differences in the convective mass flux affect the mean (domain and time aver-
 322 aged) profiles of the precipitation water mixing ratio as shown in Fig. 9. The simple micro-
 323 physics parameterization used in the simulations assumes precipitation to be in the form of

324 snow/rain in the upper/lower troposphere, with snow sedimenting with significantly smaller
325 vertical velocity. This explains the difference between lower- and upper-tropospheric val-
326 ues of each profile. However, the magnitude of the profiles (i.e., the largest/smallest for
327 SSTSP13/SSTSP31) is in direct response of the convective mass flux shown in Fig. 8. The
328 difference between precipitation water profiles might have a significant impact on model re-
329 sults once an interactive radiation scheme is used in place of a prescribed radiative cooling
330 applied in current simulations.

331 SSTSP framework significantly modifies the spatial distribution of convection and related
332 statistics. The differences in cloudiness are associated with different spatial distributions of
333 the time-averaged precipitable water content, cloud top temperature and precipitation rate,
334 as shown in Figure 10, respectively, with their mean values given in Table 1. As the figures
335 document, SSTSP simulations are characterized by significantly narrower distributions of all
336 the quantities. In the CRM simulation, central 10,000 km is characterized by the mean cloud
337 top temperature around 288 K, precipitable water around 75 kg m^{-2} , and surface precipita-
338 tion around 0.45 mm hr^{-1} . All distributions are relatively flat and feature steep gradients
339 at the edges of the warm pool with the mean precipitation dropping below 0.1 mm hr^{-1} and
340 mean cloud top temperature increasing to around 300 K. SSSTP simulations, on the other
341 hand, are characterized by narrow distributions, with peaks at the center and steep gradi-
342 ents of the mean cloud top temperature and precipitation. These differences also occur in
343 SSTSP11, that is, the SP simulation with neither time acceleration nor spatial compression,
344 and thus are a general feature of the SP simulation.

345 Because of the complicated impact of the time acceleration on diabatic processes, an in-
346 trinsic feature of the SSTSP framework, it is impossible to rescale the cloud top temperature
347 between CRM and SSTSP simulations. Increasing temporal acceleration leads more intense
348 convective activity (c.f., Fig. 8), increased cloudiness and precipitable water, and decreased
349 mean cloud top temperature (c.f., Table 1). These aspects of temporal acceleration have
350 been pointed out by Pauluis et al. (2005) and they seem related to the way microphysical

351 processes (in particular fallout of rain) are handled. No acceleration of microphysical pro-
352 cesses is applied here, potentially impacting the balance between processes responsible for
353 moistening and drying the troposphere.

354 In summary, SSTSP framework appears to simulate 2D Walker circulation qualitatively
355 well. In particular, mean large-scale flow consists of deep overturning circulation (first baro-
356 clinic mode) and mid-tropospheric jet (second baroclinic mode). Deep convection occurs
357 primarily over the warm pool, and subsidence regions are dominated by shallow convection.
358 However, detailed comparison reveals systematic differences in the model mean state. These
359 differences are mainly artifacts of the original implementation of SP, without significant
360 drawbacks of the SSTSP framework. The artificial scale separation between large-scale and
361 small-scale models and periodicity of small-scale models impose significant limitations on
362 the flow field in the small-scale domain (e.g., vanishing mean mass flux) and subsequently
363 on the simulated convection and its organization. Convective feedback to large-scales and
364 mean large conditions are modified accordingly.

365 *b. Transients*

366 SPMG document several transient features occurring in CRM simulation. The large-
367 scale flow is characterized by low-frequency variability featuring 20-day oscillations with
368 alternating periods of strong and weak overturning circulation. The strong circulation phase
369 is associated with intense convection and expansion of the large-scale convergence region
370 over the warmest SSTs. The weak circulation phase, on the other hand, features reduced
371 convective activity and narrower convergence region. The expansion/compression of the
372 convergence region coincides with synoptic-scale convective activity propagating from/to
373 the centre of the domain with the average speed between 5 and 10 m s⁻¹.

374 Here, we investigate if SSTSP framework is capable of capturing these oscillations. Fig-
375 ure 11 shows Hovmoeller diagrams of cloud top temperature for SSTSP12, SSTSP22 and
376 SSTSP32 simulations, with the CRM simulation also included for the reference. The figure

377 shows that the variability in SP simulations is of similar character to that in the CRM sim-
378 ulation. However, the zigzag pattern formed by very cold tops of convective cloud systems
379 propagating toward and then away from the convergence region apparent in the CRM simu-
380 lation is less coherent in the SP model. The coherency decreases with the increase of spatial
381 compression and temporal acceleration. Less coherent propagation of convective systems (in
382 comparison to the CRM simulation) happens even for the SP simulations with no spatial
383 compression and temporal acceleration (not shown). This is consistent with the fact that
384 coherent propagation of convective-scale features across the SP model grid is more difficult
385 than in the CRM model because cloud-scale models communicate only through the large-
386 scale model dynamics. Another feature apparent in Fig. 11 is that convection seems to be
387 more localized in the center of the domain as already documented in Figs. 10 to 11.

388 *c. Low-frequency variability*

389 Low-frequency variability in the CRM simulation has been analysed in detail in SPMG.
390 There, we apply the Empirical Orthogonal Function (EOF) analysis and develop an index of
391 the low-frequency variability. Subsequently, we construct composite of low-frequency vari-
392 ability with lag-regression analysis applying the index. We analyse reconstructed fields of dif-
393 ferent dynamical variables and describe the low-frequency oscillation. We find low-frequency
394 variability of 20 days period, triggered by anomalously intense deep convection over warm
395 pool. This, in turn, is the consequence of large-scale horizontal advection of anomalously
396 moist air from the subsidence region after the period of moisture buildup through anoma-
397 lously intense shallow convection.

398 Here, we investigate if the low-frequency variability is captured applying the SSTSP
399 framework by applying the same methodology as for the CRM simulation in SPMG. First,
400 we perform EOF analysis for the last 290 days of large-scale surface wind data with 1-
401 hour temporal resolution. Subsequently, for every SSTSP simulation, we analyze the low-
402 frequency variability applying the principal component of the leading EOF (see section 4

403 in SPMG). Table 2 presents main characteristics of the leading EOF for various SSTSP
404 simulations and for the CRM simulations from SPMG. All SSTSP simulations exhibit a
405 dominant mode of low-frequency variability corresponding to a strengthening/weakening of
406 the low level flow as identified previously for the CRM simulation in SPMG. Power spectrum
407 peaks for the period in between 23-26 days and compares reasonably well with the 20-
408 day period of the CRM simulation. It thus appears that SSTSP framework captures the
409 variability corresponding to intra-seasonal frequency band, with the variability responsible
410 for a significant percent of the total variance as in the CRM simulation. Overall, SSTSP
411 simulations with larger spatial compression or temporal acceleration tend to exhibit lower
412 total variance. SSTSP23 and SSTSP24 simulations feature the closest variance to the CRM
413 simulation.

414 In order to characterize low-frequency oscillation in more detail, composites of low-
415 frequency variability were constructed by regressing various variables on the leading EOF
416 principal component as described in SPMG. All SSTSP simulations reproduce phases of
417 the low-frequency oscillation, with the exemplary composite of the horizontal velocity for
418 SSTSP22 simulation shown in Figs. 12 and 13. Fig. 12 can be compared to Fig. 8 in SPMG,
419 whereas Fig. 15 can be compared to panels (a) in Figs. 9 to 12 in SPMG. As in SPMG, the
420 low-frequency oscillation consists of 4 phases, namely suppressed, strengthening, active and
421 decaying. The mechanism behind low-frequency oscillation is robust and it is reproduced
422 in all SSTSP simulations. As in the CRM simulation, large-scale advection of moisture is
423 correlated with oscillations of convective activity and large-scale circulation. Suppressed
424 phase is characterized by weak large-scale overturning circulation and decreasing deep con-
425 vective activity in the central part of the domain. This, in turn, is associated with drier
426 troposphere due to anomalously strong mid-tropospheric advection of dry air from the sub-
427 sidence region and anomalously weak advection of moist surface air to the central part of
428 the domain. At the same time, anomalously weak subsidence allows for moisture buildup
429 over the subsidence region, as shallow convective activity intensifies. Circulation strength-

430 ens as low-tropospheric anomalously-moist air is advected to the central part of the domain
431 and deep convection intensifies. As deep convective activity reaches its peak, troposphere
432 warms and dries due to the latent heat release and intense precipitation. The decaying
433 phase follows when the central region dries out because of the intense precipitation, and it
434 is accompanied by a strong mid-tropospheric jet bringing dry air from the subsidence region
435 and advection of anomalously dry low-tropospheric air as shallow convection weakens due
436 to strong subsidence.

437 **4. Discussion and conclusions**

438 The primary purpose of this paper is to evaluate sparse space-time superparameterization
439 (SSTSP) introduced in Xing et al. (2009). SSTSP extends the original superparameteriza-
440 tion (SP) approach, where convective processes are simulated explicitly by a cloud-resolving
441 model embedded in every large-scale model column. The motivation behind SSTSP comes
442 from the quest for computationally affordable and statistically accurate simulations of large-
443 scale scale circulations that crucially depend on convective activity. SSTSP addresses this
444 issue by significantly reducing cloud-resolving calculations and at the same time assuring its
445 statistically accurate small-scale (convective) feedback. The feedback is obtained by rescal-
446 ing statistics from cloud-resolving calculations over time- and horizontal-domain spans that
447 are reduced relatively to the large-scale time and space resolutions.

448 Xing et al. (2009) performed initial tests of the SSTSP methodology. They conducted
449 idealized simulations of a squall line within 1000-km horizontal domain for a 6-hour period.
450 It was shown that SSTSP captures propagation of a squall line across the domain, with the
451 propagation speed controlled by the prescribed shear. Here, we evaluate SSTSP framework
452 for idealized Walker cell setup. This is a more complex case than in Xing et al. (2009)
453 because it features a wider range of spatio-temporal scales, up to planetary scales and time
454 periods up to several tens of days. In contrast to Xing et al. (2009), larger-scale flow can

455 evolve in response to feedbacks from moist convection, and in turn it can affect subsequent
456 convective development. We evaluate the performance of the SSTSP algorithm by comparing
457 solutions to those obtained applying the cloud-resolving model (CRM) described in Slawinska
458 et al. (2013; SPMG).

459 We find that SSTSP is capable in reproducing key characteristics of the cloud-resolving
460 Walker cell simulation. In agreement with CRM results, the mean state features the first
461 and second baroclinic modes with deep convection over high SST organized into propagat-
462 ing systems. The properties of these convective systems (e.g., structure, propagation) are
463 affected by the horizontal extent of SP cloud-resolving domains. This is an artifact of the
464 periodicity of embedded cloud-resolving models that cannot be avoided. SSTSP captures
465 intra-seasonal variability predicted by the CRM model that consists of 4 distinctive stages
466 - suppressed, intensification, active and weakening phase - along with the mechanisms driv-
467 ing them. Differences in convective evolution and propagation result in some differences
468 between SSTSP and CRM simulations, such as in the mean sounding, spatial distribution
469 of the cloudiness and surface precipitation, or in the spatio-temporal characteristics of the
470 low-frequency oscillation. Differences in the mean cloudiness (cf. Fig. 9) will most likely be
471 accentuated when interactive radiation transfer scheme is used, an aspect not addressed in
472 the current study.

473 Numerical simulations discussed here can be put in the context of those discussed in
474 Grabowski (2001; G01 hereafter). Section 3 of G01 presented SP simulations of a 2D flow
475 driven by large-scale SST gradients, although of a significantly smaller horizontal extent
476 (computational domain of 4,000 km in G01 rather than 40,000 km here and in SPMG). SP
477 simulations in section 3 of G01 were compared to CRM simulations discussed in Grabowski et
478 al. (2000). G01's SP simulations used various horizontal gridlengths of the large-scale model
479 (from 20 to 500 km; referred to as P20 and P500, respectively) with the horizontal extent of
480 the embedded periodic-horizontal-domain CRM model matching the large-scale model gri-
481 dlength. G01 results documented a significant impact of the specific setup of the SP model

482 configuration (i.e., from P20 to P500), especially for the mesoscale convective organization
483 (cf. Fig. 8 therein). In contrast, SP simulations presented here feature just a single 48-km
484 horizontal gridlength of the large-scale model and explore the impact of SSTSP methodol-
485 ogy. Such a gridlength can be argued to follow a recommendation of Grabowski (2006a)
486 who suggested that the SP approach is better suited for large-scale models with horizontal
487 gridlengths in the mesoscale range (i.e., a few tens of km). This is because, in the mesoscale
488 gridlength case, the embedded SP models represent effects of small-scale convective motions
489 only, and the convective mesoscale organization (e.g., into squall lines) can be simulated by
490 the large-scale model. Both convective and mesoscale circulations have to be represented
491 by the SP model when the large-scale model gridlength is hundreds of km, as in typical
492 global climate applications (e.g., Khairoutdinov et al. 2005, DeMott et al. 2007, among oth-
493 ers). Section 4 of G01 applies the SP methodology to the problem of large-scale convective
494 organization on an idealized constant-SST (“tropics everywhere”) aquaplanet. Although
495 not emphasized there, the SP aquaplanet simulations (as well as subsequent studies, e.g.,
496 Grabowski 2006b) already apply the spatial compression methodology because of the dis-
497 parity between large-scale model gridlength and the horizontal domain size of the embedded
498 CRM model.

499 The obvious main drawback of the SP approach is that every cloud-resolving model is in-
500 dependent of each other and communicating solely by the large-scale model dynamics (i.e.,
501 through the large-scale forcings). The key point is that cloud systems cannot propagate
502 directly from one large-scale gridbox to the other, but they remain locked in a single large-
503 scale gridbox because of the periodic lateral boundary conditions. As discussed by Jung and
504 Arakawa (2005), periodic lateral boundary conditions require the mean mass flux to vanish.
505 As a result, updrafts get weaker as the horizontal extent of the cloud-resolving domain de-
506 creases. Large-scale thermodynamical fields are also modified, for instance, the lower/upper
507 troposphere becomes moister/drier. This key drawback of the SP (and thus SSTSP) ap-
508 proach was also noted in other studies (e.g., G01) and it is evident in our simulations.

509 To overcome these limitations, Jung and Arakawa (2005) suggest an alternative approach
510 where periodic boundary conditions are abandoned and the adjacent cloud-resolving mod-
511 els are linked allowing propagation of small-scale perturbations from one large-scale model
512 gridbox to another. Such an approach is relatively straightforward in a 2D large-scale model
513 framework, but it requires more complicated parallel-processing methods as opposed to “em-
514 barassingly parallel” logic of the original SP. Perhaps more importantly, this simple idea
515 leads to a significantly more complex methodology when implemented into a 3D large-scale
516 model (cf. Jung and Arakawa 2010). Considering these factors, we feel that the traditional
517 SP/SSTSP methodology can still serve as a valuable technique in large-scale models featur-
518 ing mesoscale horizontal gridlengths and variable orientation of SP cloud-resolving models
519 (cf. Grabowski 2004). We hope to report on such numerical experiments in forthcoming
520 publications.

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TABLE 1. Mean precipitable water content (prw; in kg m^{-2}), cloud top temperature (cltop; in K) and precipitation (precip; in mm hr^{-1}).

$p_x p_t$	prw	cltop	precip
11	53.8	297.3	0.128
12	54.2	294.4	0.130
13	54.6	291.9	0.133
14	54.8	290.0	0.133
21	51.0	297.3	0.122
22	51.1	294.5	0.123
23	51.4	292.4	0.125
24	51.1	290.5	0.125
31	47.0	297.5	0.108
32	47.2	295.2	0.110
33	47.3	293.0	0.111
34	47.6	291.3	0.111
CRM	55.9	296.5	0.157

TABLE 2. Eigenvalue periods of the first EOF for the surface wind time series and standard deviation of its principal component. Simulations with SSTSP algorithm (with p_x and p_t as given in the first column). CRM results are included in the bottom row.

p_x p_t	eigenvalue	period (days)	standard deviation
11	0.51	24	157
12	0.53	26	154
13	0.54	24	159
14	0.42	24	133
21	0.41	26	117
22	0.29	24	85
23	0.39	26	108
24	0.33	26	95
31	0.29	24	86
32	0.22	23	71
33	0.18	23	62
34	0.18	26	63
CRM	0.36	20	88

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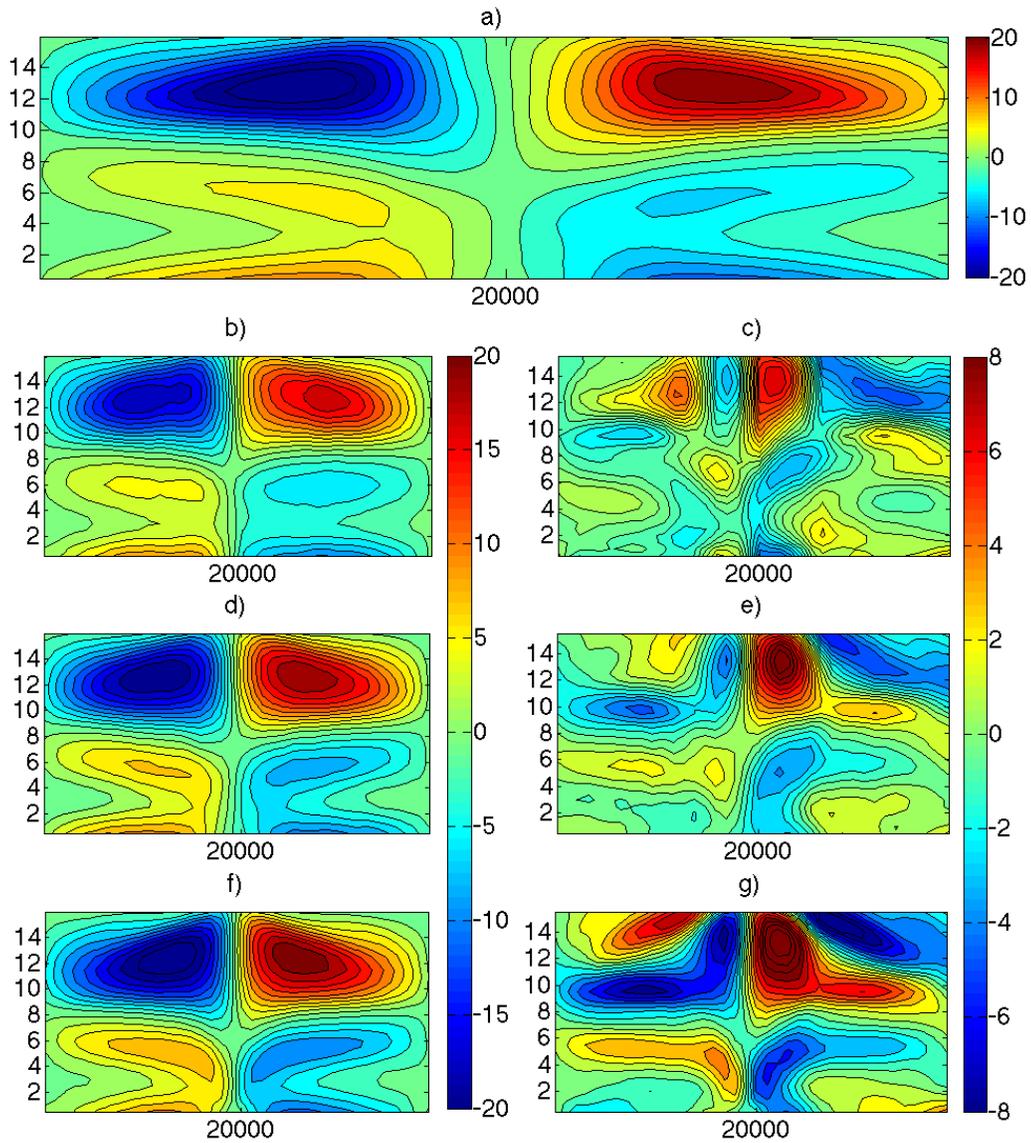


FIG. 1. a) Time averaged horizontal velocity field (m s^{-1}) for CRM. b-g) Time averaged horizontal velocity field (m s^{-1}) for SSTSP11/SSTSP22/SSTSP33 (top/middle/bottom left) simulations and their difference from the CRM simulation (top/middle/bottom right).

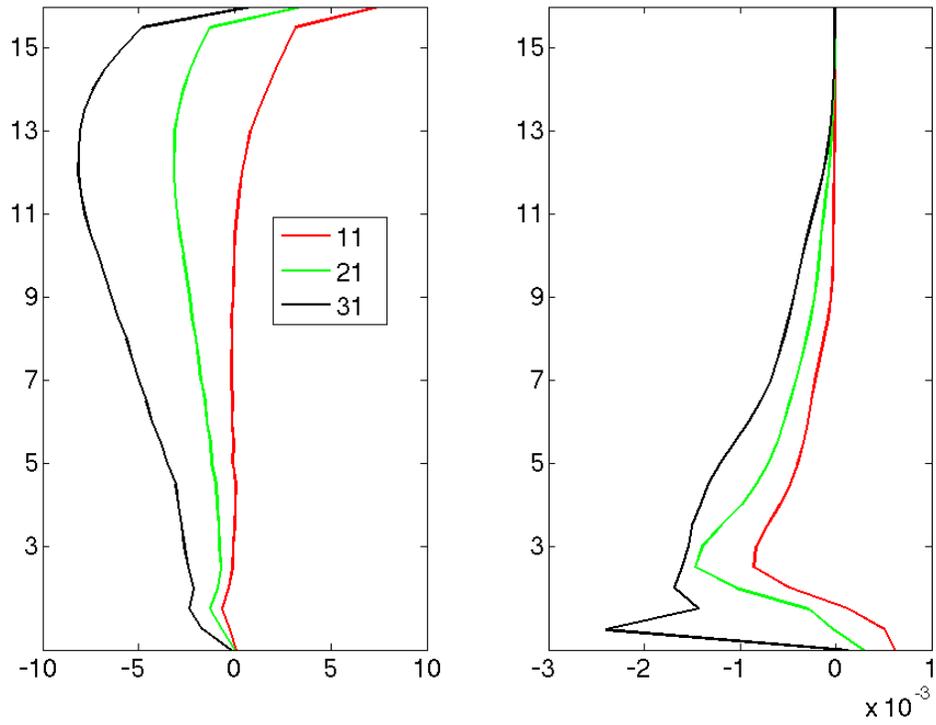


FIG. 2. Difference in the mean profiles of the potential temperature (K, left) and the water vapour mixing ratio (kg kg^{-1} , right) between SSTPS11/SSTSP21/SSTSP31 (red/green/black line) and the CRM simulation.

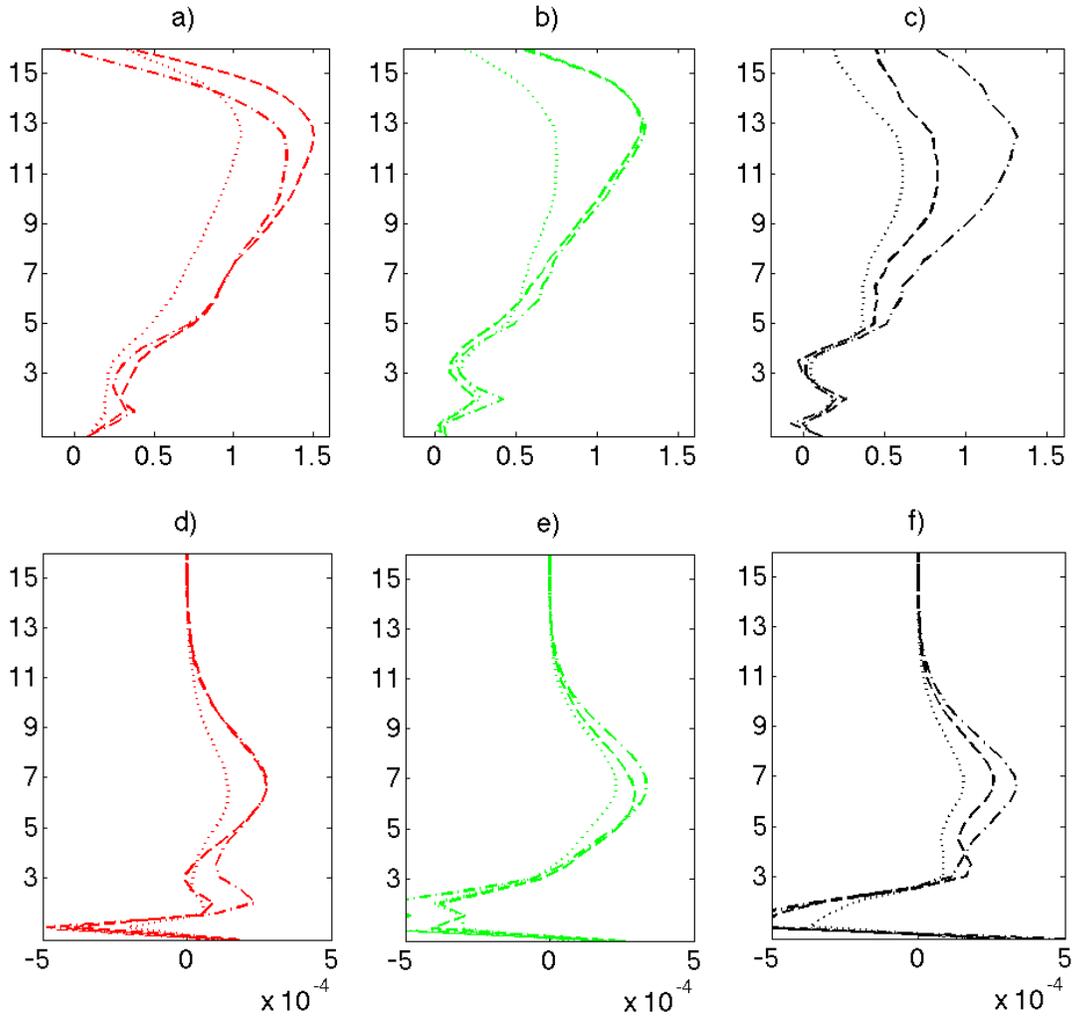


FIG. 3. Difference in mean profiles of the potential temperature (K, top) and the water vapour mixing ratio (kg kg^{-1} , bottom) between SSTSP11/SSTSP21/SSTSP31 (left/middle/right) and SSTSP simulation with the same spatial acceleration p_x and temporal acceleration $p_t=2/3/4$ (dotted/dashed/dashed-dotted lines), respectively.

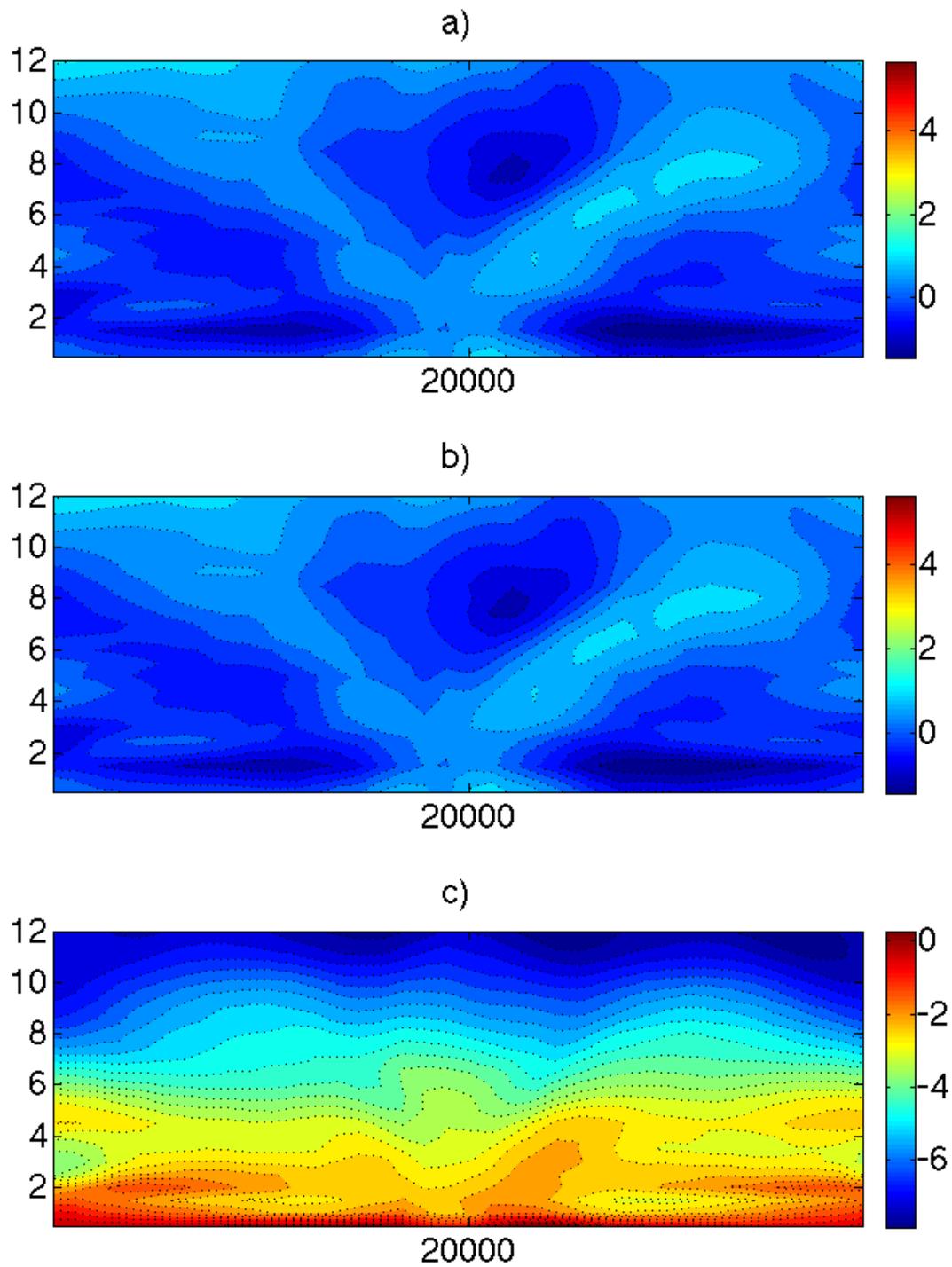


FIG. 4. Spatial distribution of the difference in the time averaged potential temperature field (K) for the SSTSP11/SSTSP22/SSTSP33 (top/middle/bottom) simulation and the CRM simulation.

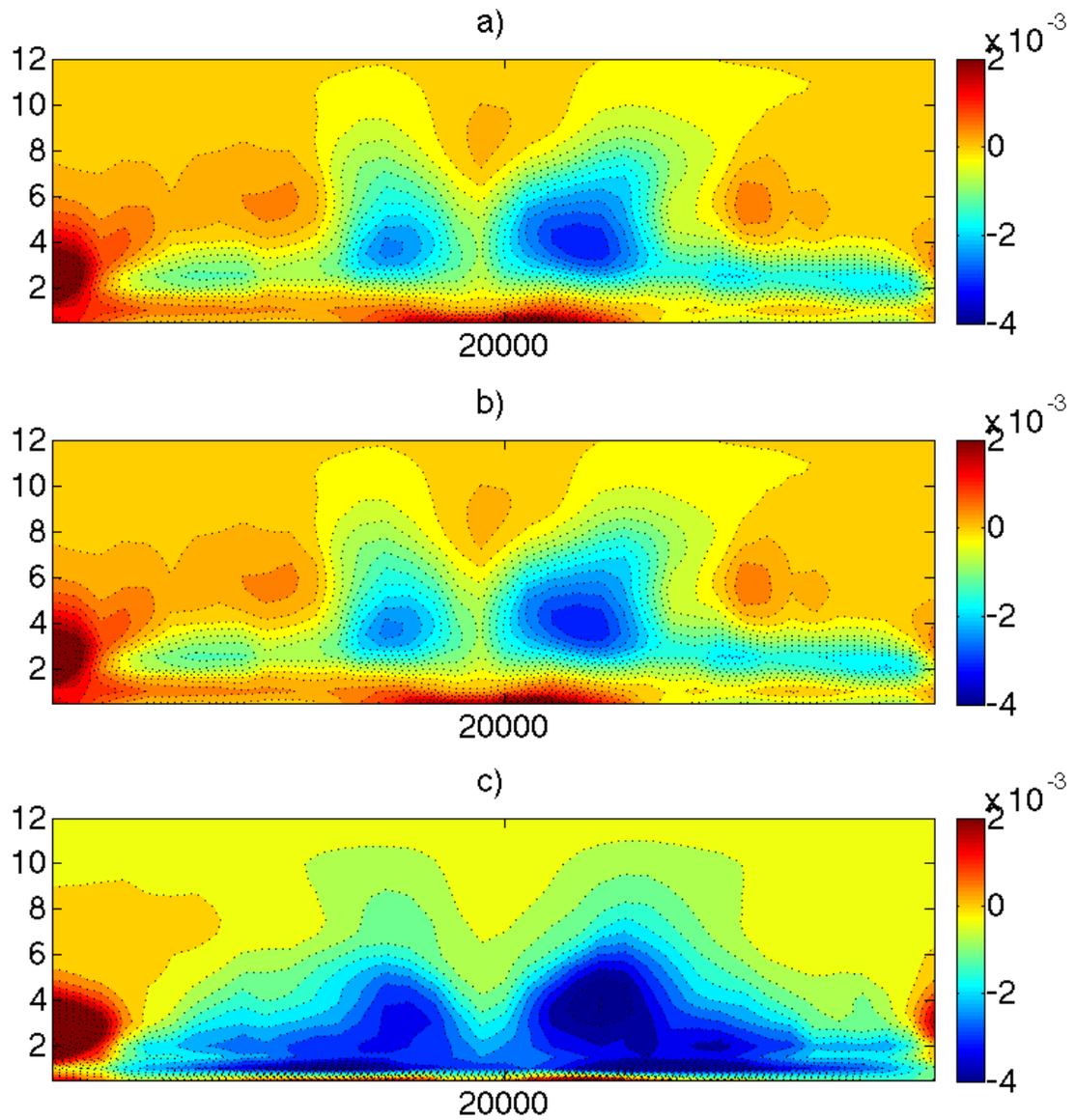


FIG. 5. Spatial distribution of the difference in the time averaged field of the water vapor mixing ratio (kg kg^{-1}) for the SSTSP11/SSTSP22/SSTSP33 (top/middle/bottom) simulation and the CRM simulation.

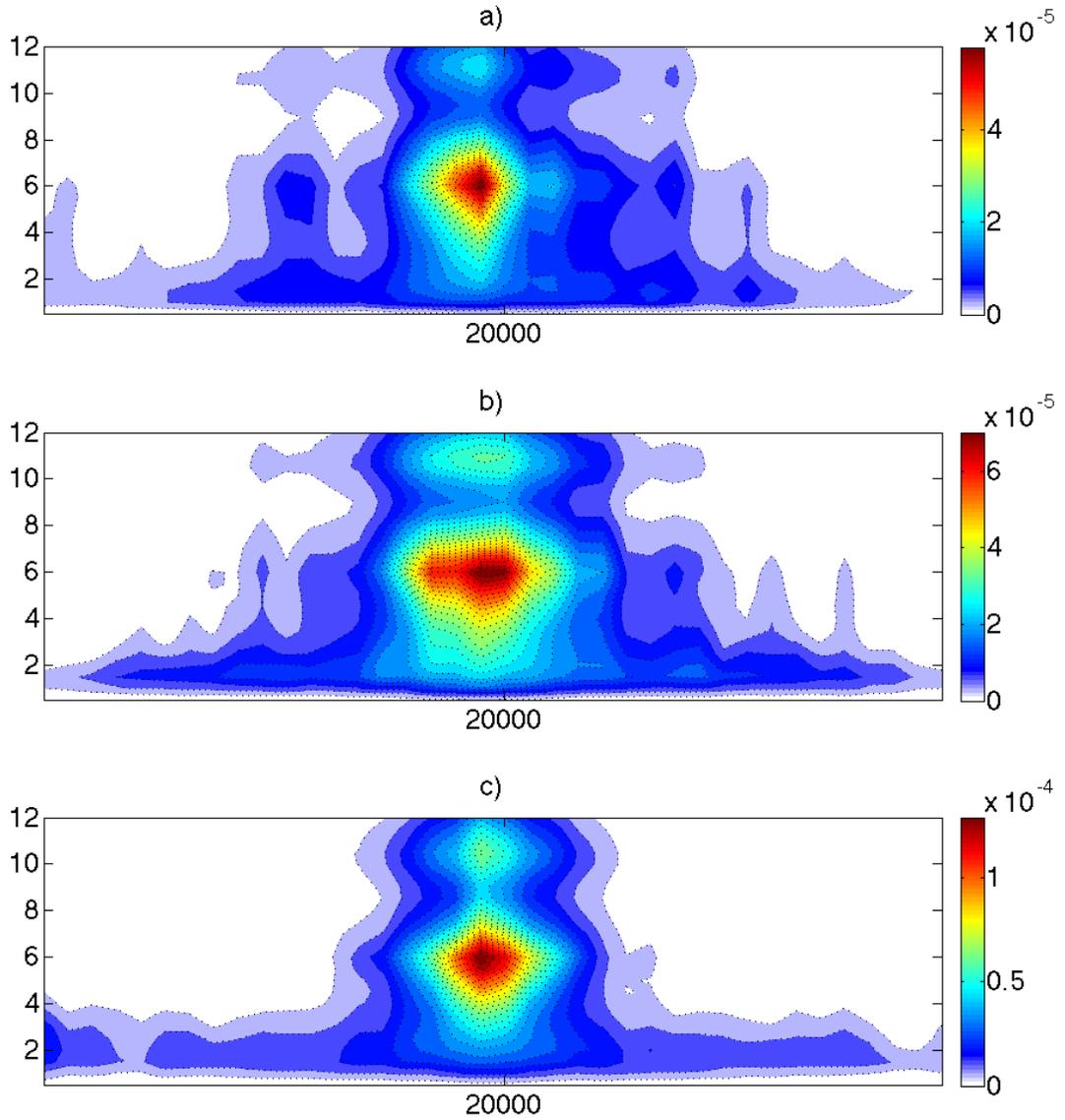


FIG. 6. Spatial distribution of the time averaged cloud water mixing ratio (kg kg^{-1}) for SSTSP11/SSTSP22/SSTSP33 (top/middle/bottom) simulations.

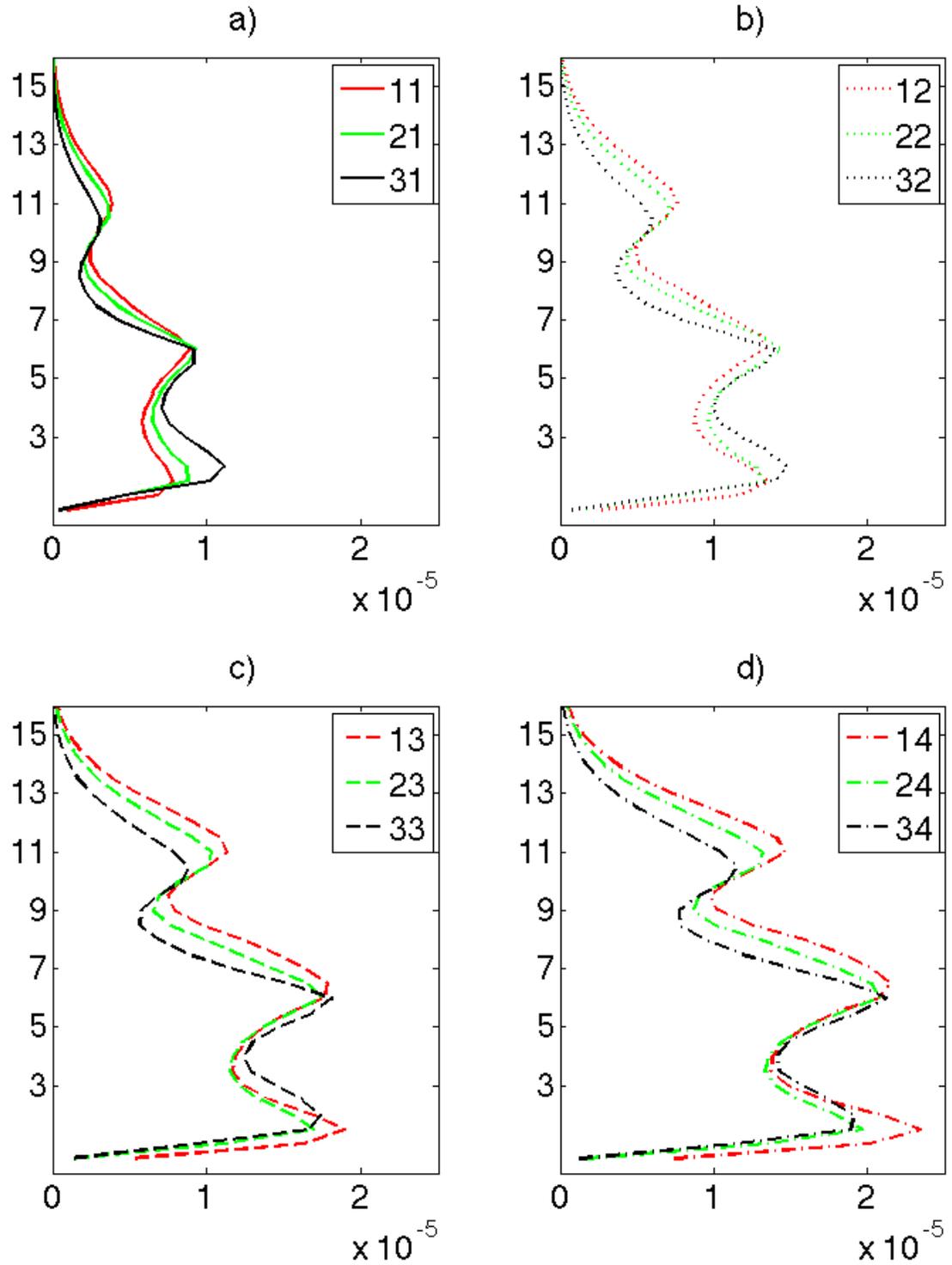


FIG. 7. Mean (time and space averaged) profiles of the cloud water mixing ratio (kg kg^{-1}). Red/green/black lines are for simulations with spatial compression $p_x=1/2/3$, respectively. Solid/dotted/dashed/dashed-dotted lines are for simulations with temporal acceleration $p_t=1/2/3/4$, respectively.

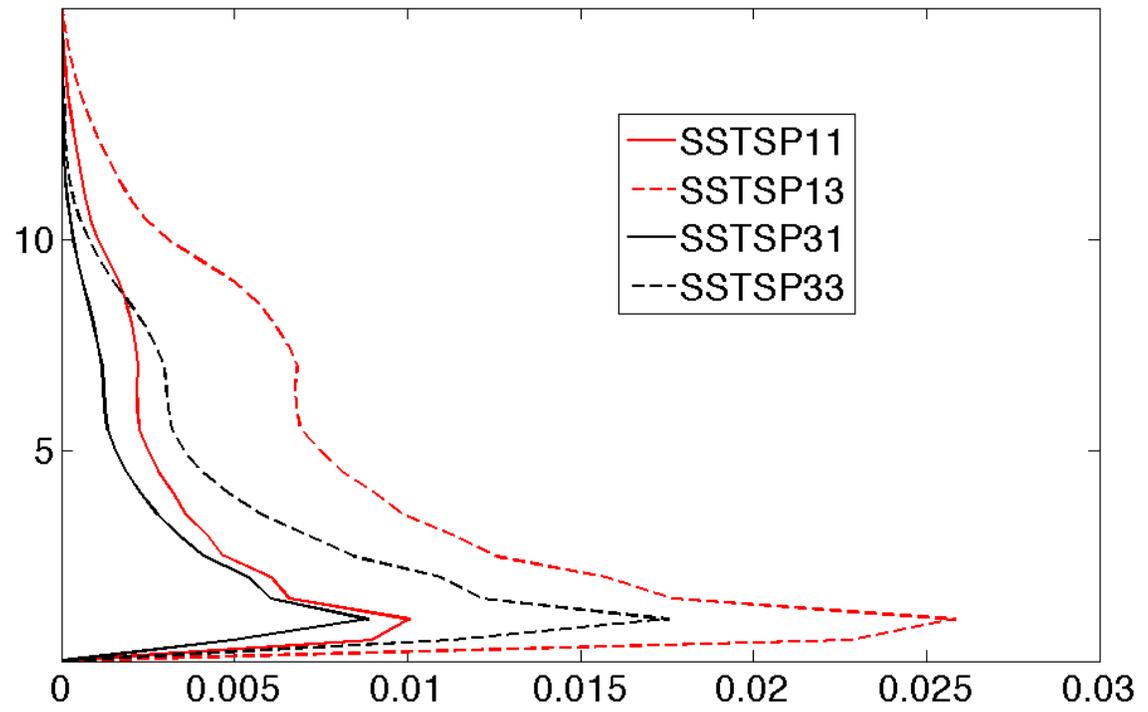


FIG. 8. Mean (time and space averaged) mass flux profiles for simulation with spatial compression 1 or 3 (red or black line) and temporal acceleration 1 or 3 (solid or dashed line).

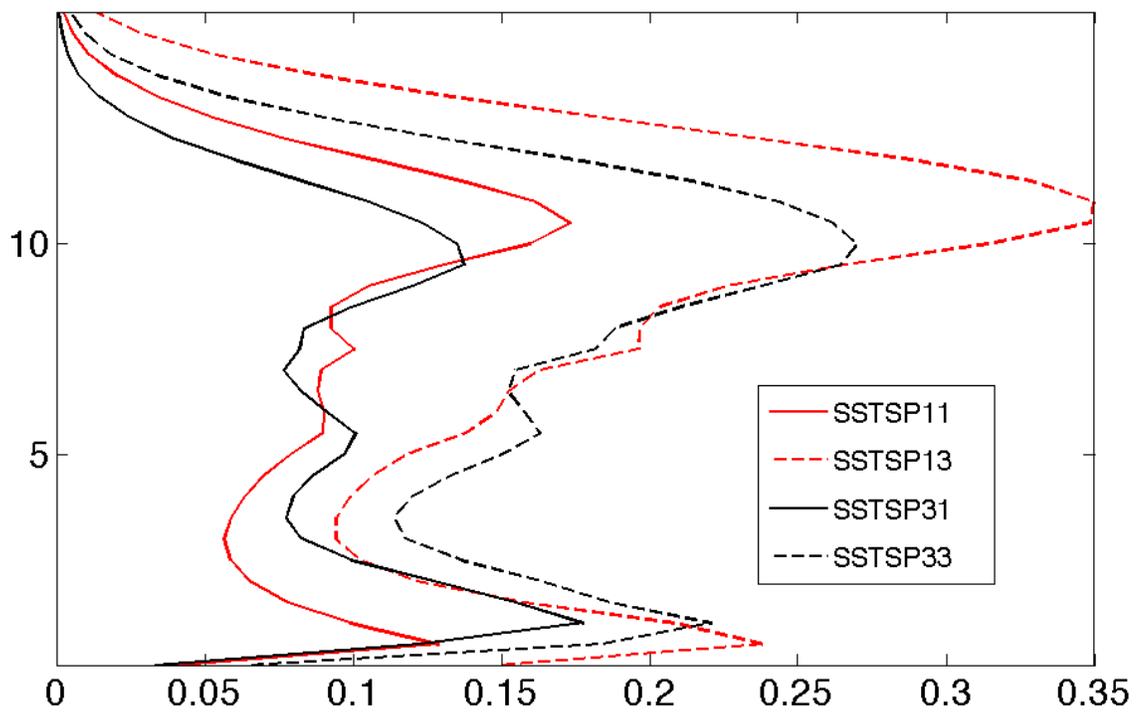


FIG. 9. As Fig. 8, but for the precipitation water water mixing ratio (kg kg^{-1}).

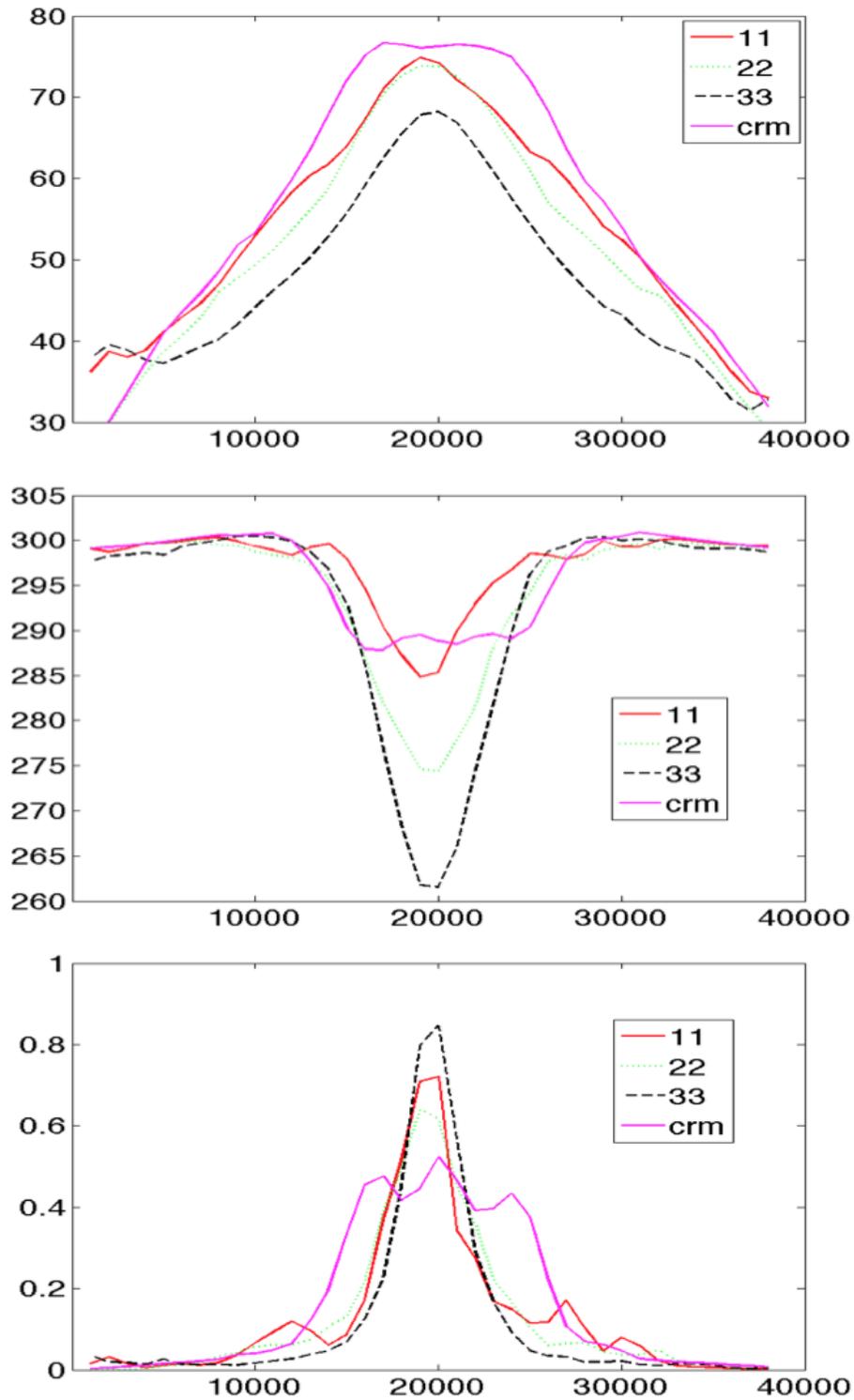


FIG. 10. Time averaged horizontal distribution of precipitable water (kg m^{-2} , top), cloud top temperature (K middle) and precipitation (mmh^{-1} , bottom), for CRM (solid magenta line), SSTSP11 (solid red line), SSTSP22 (dotted green line) and SSTSP33 (dashed black line). Mean values for all simulations are given in Table 1.

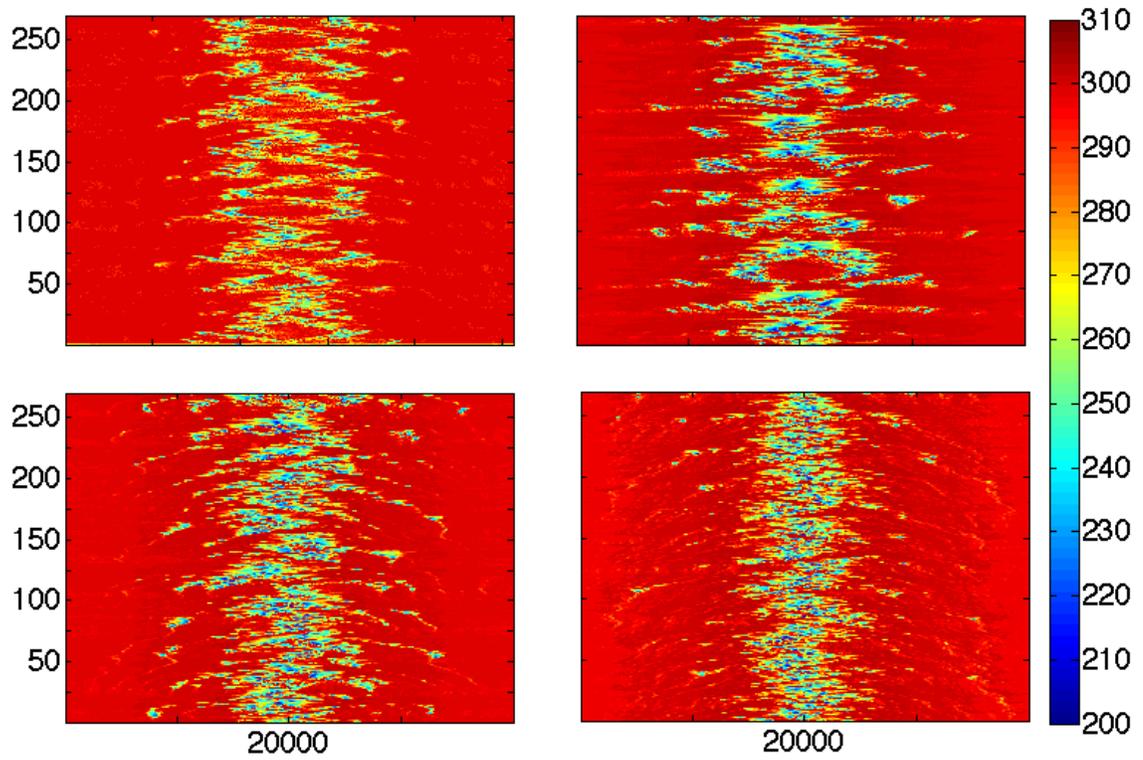


FIG. 11. Hovmoller diagrams of the cloud top temperature for CRM (top left), SSTSP12 (top right), SSTSP22 (bottom left) and SSTSP32 (bottom right) simulations.

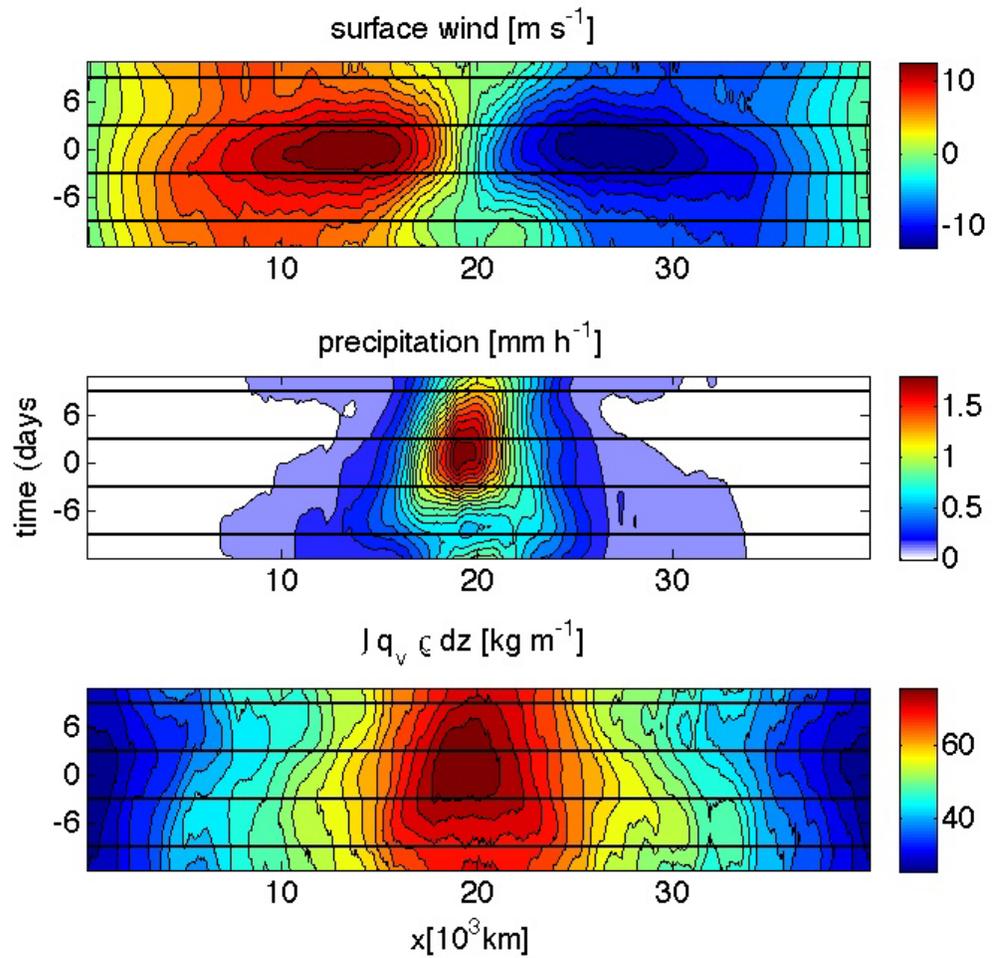


FIG. 12. Hovmoller diagrams of the lag regression of the surface wind (m s^{-1} ; upper panel), surface precipitation rate (mm h^{-1} ; middle panel), and precipitable water (kg m^{-2} ; bottom panel) for SSTSP22 simulation.

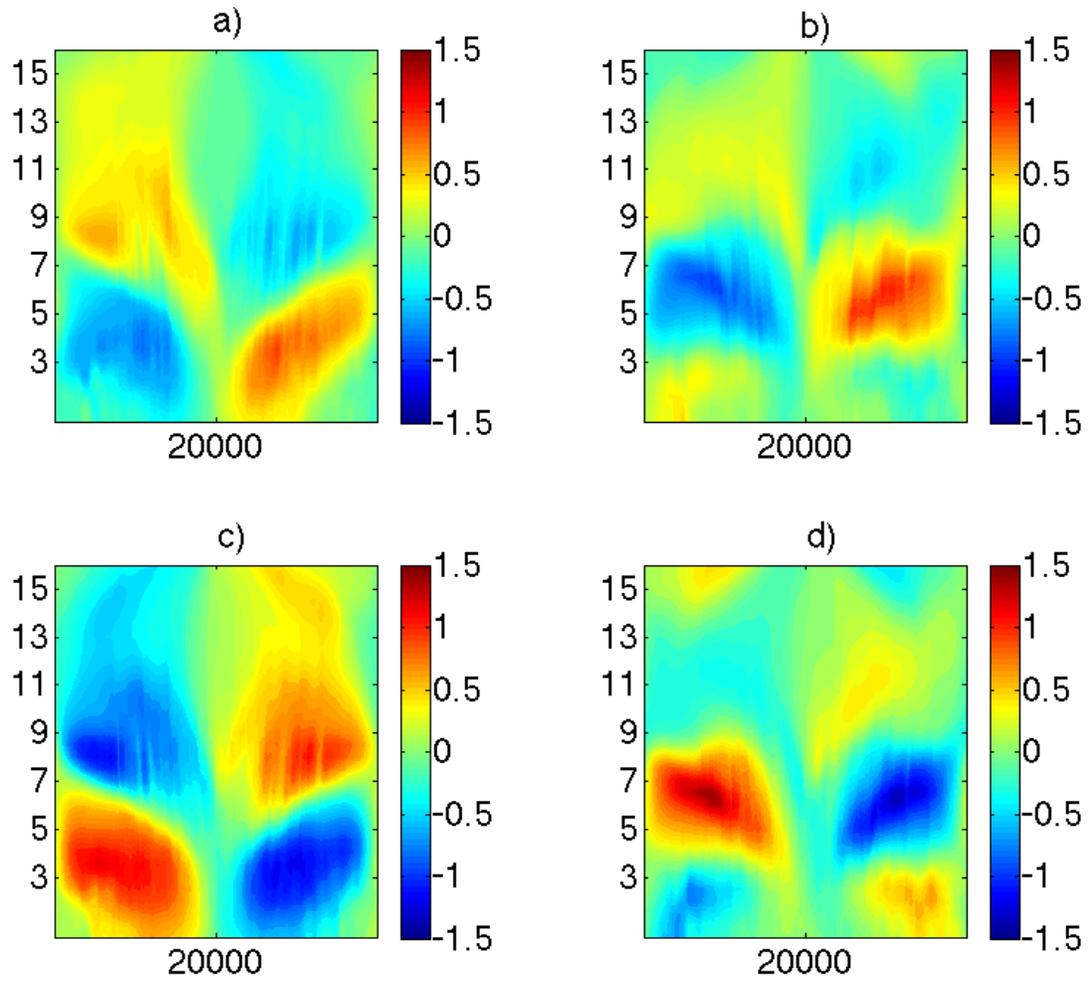


FIG. 13. Lag-regressed structure of the horizontal velocity anomaly for (a) suppressed phase, (b) strengthening phase, (c) active phase, and (d) decaying phase for SSTSP22 simulation.