Simple multicloud models for the diurnal cycle of tropical precipitation. Part II: The continental regime

YEVGENIY FRENKEL

Courant Institute for Mathematical Sciences, New York University, New York, NY

BOUALEM KHOUIDER *

Department of Mathematics and Statistics, University of Victoria, Victoria, BC, Canada

ANDREW J. MAJDA

Centre for Atmosphere-Ocean Sciences and Courant Institute, New York University, New York, NY.

*Corresponding author address: Boualem Khouider, Department of Mathematics and Statistics, University of Victoria, 3800 Finnerty Road, Victoria, British Columbia, Canada.

E-mail: khouider@uvic.ca
ABSTRACT

The variation of precipitation over land due to the diurnal cycle of solar heating is examined here in the context of a simple multicloud model for tropical convection with bulk atmospheric boundary layer (ABL) dynamics. The model utilizes three cloud types, congestus, deep, and stratiform that are believed to characterize organized tropical convection and based on the first two baroclinic modes of vertical structure in the free troposphere, coupled to the ABL through full bulk boundary layer (FBBL) dynamics that allow a careful separation between sensible and latent heat surface fluxes. In a land parameter regime, characterized by a strong inversion profile, a large Bowen ratio of 0.4 and active mixing of sensible heat due to cumulus entrainment and downdraft fluxes at the top of the ABL, the model supports a stable one-day periodic solution that is characterized by a pronounced (7 K/day) afternoon peak in precipitation consistent with observations of tropical precipitation over continental regions. The current study suggests a division of the diurnal cycle of precipitation over land into a cycle of five phases: (1) an overnight phase of radiative convective equilibrium state between 8:00 pm and 6:00 am; (2) an early morning CAPE build up accompanied by a sudden rise in precipitation that quickly dries the middle troposphere occurs between 6:00 am and roughly 10:00 am; (3) a moistening phase between roughly 10:00 am and 4:00 pm; (4) a phase of maximum precipitation between 4:00 pm and 6:00 pm which dries the middle troposphere and quickly consumes CAPE; (5) a rapid re-moistening phase which restores the moisture level to sustain the overnight RCE precipitation and connects to phase (1) in the cycle. Sensitivity tests in the model confirm that the late afternoon precipitation maximum over land depends crucially on the strong inversion, the large Bowen ratio and the active mixing of sensible heat due to cumulus entrainment and downdraft fluxes at the top of the
1. Introduction

As motivated in Part I (Frenkel et al. 2010, hereafter Part I), the diurnal variation of tropical precipitation involves three different regimes of behavior depending on geographic location (Kikuchi and Wang 2008). An oceanic regime characterized by an early morning precipitation maximum of moderate strength, a continental regime characterized by a strong precipitation peak occurring in the afternoon, and a more complex coastal regime that involves phase propagation of the precipitation maximum towards land and/or towards the deep ocean basin depending on the time of the day. As pointed out in Part I, we are interested in modeling the main physical mechanisms that drive the two different regimes of diurnal cycle of tropical precipitation over the ocean and over land, respectively, which remain poorly understood and which present a challenge for current general circulation models (Randall et al. 1991; Yang and Slingo 2001; Dai and Trenberth 2004; Tian et al. 2004; Yang and Smith 2006; Kikuchi and Wang 2008). While Part I focused on the simplest model for the oceanic regime, here the diurnal cycle over land is the main emphasis. The strategy here consists of using the simple multicloud model for organized tropical convection (Khouider and Majda 2006, 2008b, hereafter KM06, KM08) with more elaborate boundary layer dynamics, which among other things allows for a separate treatment of sensible and latent heat surface fluxes and explicit representation of shallow cumulus clouds. Such model was introduced and studied by Waite and Khouider (Waite and Khouider 2009, hereafter WK09) without the effects of the diurnal cycle.
The multicloud models are complex enough to capture most of the subtle interactions between different cloud-types in the tropics and therefore provide the first prototype cumulus parametrization capable of representing multi-scale organized tropical convective systems including convectively coupled waves (Khouider and Majda 2006, 2007, 2008a,b) and the Madden-Julian oscillation (Khouider et al. 2010). Yet, these parameterizations are simple enough to be able to track down and understand the physical mechanisms at work in the model. Here we take advantage of this feature to understand the physical mechanisms that drive the diurnal cycle of tropical precipitation over land. As in Part I, the main idea is to look for stable one-day periodic solutions that exhibit similar features as in observations when the model is forced by a surface (latent and sensible) heating that mimics the diurnal cycle of solar heating, under parameter regimes that characterize the tropical climatology over land. The technical procedure and methodology together with the ocean parameter regime are reported in Part I while the land regime is presented here. Due to the importance of sensible heat fluxes for the diurnal cycle of precipitation over land, only the version of the multicloud model with full bulk boundary layer dynamics is considered here. This is unlike the ocean case considered in Part I where the original multicloud model, for which the boundary layer dynamics are reduced to a single equation for the equivalent potential temperature, is sufficient to explain the diurnal cycle of precipitation over the ocean, due to the small Bowen ratio that. Nevertheless, it was established in Part I that the model with full boundary layer dynamics (WK09) gave essentially the same mechanism for diurnal cycle over the ocean as the simpler model (KM08).

The present paper is organized as follows. In section 2, we briefly review the Waite-Khouider (WK) model which was discussed in some detail in Part I. Here the review empha-
sizes the diurnal variations of sensible and latent heat surface fluxes and the use of a strong temperature inversion above atmospheric boundary layer (ABL) that are important for the land regime. The mathematical methodology used here to search for one-day periodic solutions is the same as described in the Appendix of Part I. In section 3, we present the main results, for the land case, consisting of one-day periodic stable solution characterized by a strong afternoon precipitation peak consistent with observations, and discuss the physical mechanisms underlying this solution. In section 4, we present sensitivity tests to highlight the physical parameters that control the diurnal cycle of precipitation over land. Finally, a concluding summary and discussion is given in section 5.

2. Formulation of WK model with strong inversion

a. Governing equations

Two version of the multicloud model (KM06,KM08) were presented and used in part I (Frenkel et al. 2010) to simulate the diurnal cycle of precipitation over the tropical oceans, one with the boundary layer dynamics reduced to a single equation for the equivalent potential temperature (averaged over the ABL depth) and one that uses a full bulk boundary layer model following Stevens (2006) that, among other important details, separates latent and sensible heat surface fluxes and has an explicit representation of shallow cumulus clouds, namely the WK model (WK09).

We recall that the multicloud models (KM06,KM08,WK09) assume three heating profiles associated with the three main cloud types that are observed to characterize organized
tropical convective systems (Johnson et al. 1999): Cumulus congestus clouds with heating below and cooling aloft, deep convective towers that heat the whole tropospheric depth, and lagging-stratiform anvils that heat the upper troposphere and cool the lower troposphere. Their dynamical core consists of the momentum and potential temperature equations for the first and second baroclinic modes of vertical structure coupled to the boundary layer (KM06, KM08, WK09). The multicloud models carry an equation for the vertically averaged moisture and use a non-linear switch function to make systematic transitions between dry tropospheric regimes where deep convection is inhibited while congestus clouds are promoted and moist preconditioned states allowing for deep convection.

Although the WK model was presented and discussed in some detail in Part I, the model equations are reported in Table 1 and the associated constants and parameters are reported in Table 2, for the sake of completeness. All equations in Table 1 are given in non-dimensional form where the speed of the first baroclinic Kelvin waves, \( c_r \approx 50 \text{ ms}^{-1} \), is the velocity scale, the equatorial Rossby radius of deformation, \( L_e \approx 1500 \text{ km} \), is the length scale, \( T = L_e/c_r \approx 8.33 \text{ hours} \) is the time scale, and \( \bar{\alpha} = H_T N^2 \theta_0 / \pi g \approx 15 \text{ K} \) is the temperature unit scale. It is also important to recall that, in Part I, some crucial changes were introduced into the WK model in order to allow for the exchange of sensible heat fluxes between the ABL and the deep troposphere through shallow cumulus entrainment and downdraft fluxes; entrainment and downdraft parameters \( \alpha_E \) and \( \beta_D \) for these effects were introduced into the free troposphere moisture and first baroclinic temperature equations in Table 1. These effects were absent in the original WK model (WK09) since sensible heat fluxes from the boundary are believed to be negligible for tropical convection over the ocean; however, for explaining diurnal cycle over land, these modifications are essential here.
The effects of the diurnal cycle of solar heating on the ABL are represented through the surface fluxes $\Delta_s q = q_s - q_b$ and $\Delta_s \theta = \theta_s - \theta_b$ of moisture and temperature, in the form of one-day periodic perturbations in the form of a half-sine function (see Part I and (2) below) added to the surface values $q_s$ and $\theta_s$, respectively.

$$q_s(t) = \bar{q}_s + \hat{q}_s(t), \quad \theta_s(t) = \bar{\theta}_s + \hat{\theta}_s(t)$$

(1)

where $t$ is time.

To accommodate strong temperature inversions that characterize continental regions in the tropics (Fu et al. 1999; Fisch et al. 2004), we assume a cooler boundary layer at RCE (consistent with nocturnal radiative cooling) while preserving the equilibrium temperature profile in the free troposphere (i.e dry lapse rate) identical to the case over the ocean where the temperature inversion is weak or non-existent (see Fig. 1). Here we use $\Delta_t \bar{\theta} = -7.5$ K as our standard value for the temperature inversion over land, which is sufficient in magnitude to produce the desired afternoon peak in precipitation with a reasonable strength.

The equilibrium profiles of $\theta$ and $\theta_e$ used here for the land regime are plotted with dark lines in Fig. 1, on the left and right panels, respectively while the light lines represent the ocean reference profiles. It should be noted that in the original model the discrepancy between boundary layer and mid-tropospheric potential temperature ($\Delta_m \bar{\theta} = \bar{\theta}_b - \bar{\theta}_m$) is constrained by the value of $\gamma \equiv -\Delta_m \bar{\theta} / \Delta_m \bar{\theta}_e$ given in Table 2. To preserve the original free troposphere lapse rate ($X_t - X_m$) for both the moisture and potential temperature, the boundary layer potential temperature is cooled down to match the temperature inversion on top of the ABL. This results in $\Delta_m \theta_e = 6.5$ K for the standard case, which is in the range of observed values over the Amazon forest (Fu et al. 1999). Furthermore, $\Delta_s \bar{\theta}$ (which is set to
zero over the ocean) is lowered by approximately the same amount as the inversion which is consistent with the large Bowen ratios observed over land.

We recall that given the climatological values for the discrepancies $\Delta_x \bar{\phi}$, $x = s, m, t$ (for both $\phi = \theta$ and $\phi = q$ with $s$, the surface, $m$, the middle troposphere and $t$, the top of the boundary layer) and the rate of radiative cooling $Q_{R,1}^0$, the homogeneous RCE solution determines the values of the constants $Q_{R,2}^0, Q_{R,b}, \bar{Q}, m_0$, and the surface evaporation time scale $\tau_e$ (WK09). As a consequence of the strong temperature inversion at the top of the ABL, the value of $\tau_e$ in the present-land setup, turns out to be around two hours. This may seem somewhat alarming because it is much smaller than the typical eight hours reference value used for the ocean case in Part I and in (KM06,KM08,WK09). However, according to similarity theory this decrease in the $\tau_e$ value can be associated to a larger roughness length-scale over land terrain (Stull 1988). Additionally, the updraft mass flux velocity at the top of the ABL ($M_u$), assumed to be proportional to the convective downdraft $D_c$ (Raymond 1995), is smaller consistent with the inversion dominated subsidence regime. This is accomplished by increasing the value of the parameter $\alpha_m$ from 0.2 to 0.7 ($\alpha_m = D_c/M_u$). Overall, physically motivated choices of both $\alpha_m$ and $\Delta_s \bar{\theta}$ allow for a reasonable ABL cooling rate and evaporation time scale at RCE. While the choice of the parameters is not unique, the qualitative behaviour of the system presented below is quite robust as discussed in section 4.

As pointed out above, the value of $\Delta_m \bar{\theta}_e$ is lower due to the strong $\theta$ inversion as can be seen in Fig. 1. One critical variable that depends directly on the parameter $\Delta_m \bar{\theta}_e$ is the nonlinear switch $\Lambda$, which controls the transition between congestus and deep convection (KM06,KM08). To account for this drop in the standard value of $\Delta_m \bar{\theta}_e$, without disturbing
the model physics, we lower the threshold values of $\Delta_m \theta_e$ that define the extremely dry and extremely moist limits by the amount of the inversion value yielding, in the standard case, $\theta^+ = 12.5$ K and $\theta^- = 2.5$ K, respectively.

In addition to the changes in inversion and boundary layer cumulus updrafts, the present version of the WK model has two other important new modifications. Firstly, the congestus rain fraction is set to zero, since congestus clouds are rarely observed to precipitate over land (Casey et al. 2007). Secondly, the dry convective buoyancy parameter is doubled to $a_0 = 14$ to make deep convection more sensitive to variations in deep-troposphere potential temperature. The model’s sensitivity to changes in those and other important parameters is discussed in section 4.

b. **Diurnal cycle forcing, Bowen ratio and Galilean transformation**

While diurnal variability over the ocean is driven by latent heat, over land it is due to both latent and sensible heat. Different heat capacities of land and ocean also influence the amount of energy available for the diurnal cycle at a given time of the day but we neglect this effect here for the sake of simplicity. A useful quantity in the study of the diurnal cycle forcing over land is the ratio of fluxes of sensible heat and latent heat (produced by solar heating) known as the Bowen ratio and denoted here as $\beta$ (not to be confused with the gradient of the Coriolis force which is ignored here). Over the ocean, the Bowen ratio is typically less than 0.1 (Hsu 1998; Sadhuram et al. 2001) due to the fact that latent heat is the primary response of the ocean surface to solar heating. This is exploited in the original derivation of the multicloud model by Khouider and Majda (KM06, KM08) where a single
equation for the ABL $\theta_e$ was used to represent the boundary layer dynamics. Over desert regions, the Bowen ratio can reach values of up to $\beta = 10$ (Chapin et al. 2002). The scarcity of moisture and high surface albedo makes sensible heat the dominant response of the desert terrain to solar heating. In this paper, we are mostly concerned with the case of equatorial forests where the large transpiration and soil moisture allows for Bowen ratios in the range of $\beta = 0.2$ to $\beta = 0.4$ (Chapin et al. 2002).

In the present model, as shown in (1), the effect of solar radiation is represented through perturbations of both the surface temperature and surface moisture variables. Here the amplitude of the diurnal variation in surface temperature is inferred directly from observations (Yang and Slingo 2001) while a Bowen ratio that is appropriate for the Amazon forest (Chapin et al. 2002) is used to approximate the corresponding moisture surface flux perturbation. Accordingly, the perturbations of surface moisture and temperature used in the present study have a maximum amplitude of 10 K and 4 K, respectively. While the Bowen ratio is appropriate for the equatorial forest, the amplitude of the variation is deliberately conservative. In fact, it makes variations in the boundary layer equivalent potential temperature comparable to the perturbation used in the ocean case addressed in Part I to facilitate comparison. It should be further noted that the perturbation used in the ocean case was calculated from near maximum value of observed SST diurnal variation while the solutions with smaller perturbation showed the same qualitative behavior. In the present case, we choose the amplitude of the perturbation conservatively to reduce the set of parameter changes when going back and forth between the ocean and land scenarios. In the continental regime, the results presented in this paper are qualitatively similar to the cases with higher perturbation amplitudes. In fact, higher total perturbations produce higher
afternoon precipitation maxima with similar cycle, see section 4.

As in Part I, the same half sine perturbation profile is used here for both surface latent and sensible heat diurnal variations. It can be justified by considering observed sensible and latent heat flux where the small effects of soil thermal inertia are ignored.

\[
\dot{\theta}_{eb}(x, t) = \dot{\theta}_{eb,max} \begin{cases} 
\frac{\pi}{\pi-1} (\sin(x + ct) - \frac{1}{2}) & \text{if } x + ct \mod(2\pi) < \pi \\
-\frac{1}{\pi-1} & \text{if } x + ct \mod(2\pi) > \pi
\end{cases}, \tag{2}
\]

Here we recall the Galilean transformation that is applied to the governing equations to facilitate the analysis. The peak of solar energy traverses the 40,000 km perimeter of Earth every 24 hours, with constant speed of \( c = 463 \) m s\(^{-1}\). This naturally leads to a reference frame aligned with this constant velocity motion. A Galilean transformation \( x_g = x + ct, \tau = t \) is thus applied to the governing equations where the effects of waves and other non-homogeneities are ignored (Part I). Notice that accordingly, variations in \( x_g \) can be interpreted as relating to different locations on the equator during the same time of the day or relating to a fixed location at different times of the day, i.e local solar time (LST). The latter is adopted throughout the paper just like in Part I. To focus on the diurnal variation of the background climatology, without consideration of wave activity and other flow complexities, we seek solutions \( U = U(x_g) \) that do not depend on the new time variable \( \tau \). This results in an ODE system with periodic forcing as reported in Part I. This system is reproduced here for the sake of completeness.
\begin{verbatim}
\text{cu}' = -\bar{u}u' - p_0' - \bar{u}'(\bar{u} - u_b) + \frac{E_u}{H_T} \Delta_t u \\
\text{cu}_j' = -\bar{u}u_j' - u_j \bar{u}' - \theta_j' - u_j/\tau_R + \frac{\sqrt{2}}{T_T} \delta_b \Delta_t u \\
\text{c}\theta_1' = -\bar{u}\theta_1' + u_1' - \sqrt{2}\bar{u}' + \frac{\pi}{2\sqrt{2}} (P + \alpha E) \Delta_t \theta + \beta_D (\frac{M_d}{H_t} + \bar{u}') \Delta_m \theta - Q_{R1} - \theta_1/\tau_D \\
\text{c}\theta_2' = -\bar{u}\theta_2' + \frac{1}{4} u_2' - \frac{\sqrt{2}}{4} \bar{u}' - H_s + H_c - Q_{R2} - \theta_2/\tau_D \\
\text{c}q' = -\bar{u}q - [(u_1 + \delta u_2)q + (u_1 + \lambda u_2)\tilde{Q} - \bar{u}\tilde{Q}_0]' - P \\
+ \frac{\bar{u}}{H_T} \Delta_t ((1 - \alpha E) \theta + q_b) + (\frac{M_d}{H_t} + \bar{u}') \Delta_m ((1 - \beta_D) \theta + q_b) \\
\text{c}q_b' = -u_b q_b' - \frac{E}{h_b} \Delta_t q - \frac{M_d}{h_b} \Delta_m q + \frac{1}{\tau_e} \Delta_s q \\
\text{c}\theta_b' = -u_b \theta_b' - \frac{E}{h_b} \Delta_t \theta - \frac{M_d}{h_b} \Delta_m \theta + \frac{1}{\tau_e} \Delta_s \theta - Q_{Rb} \\
\text{cu}_b' = -u_b u_b' - p_b' - \frac{E_u}{h_b} \Delta_t u - \frac{C_d U}{h_b} u_b. \\
p_0 = p_b + \sqrt{2}(\theta_1 + \theta_2) + \frac{\pi}{2} \delta_b \theta_b \\
(1 + \delta_b)p'_b = 2\delta_b \bar{u}'(\bar{u} - u_b) - \sqrt{2}(\theta_1' + \theta_2') - \frac{\pi}{2} \delta_b \theta_b' - \delta_b \frac{C_d u_0}{h_b} u_b
\end{verbatim}

Here the prime denotes the derivative with respect to the variable \( x_g \), which is the only independent variable in this system while the dependent variables are as in Table 1. To be more explicit, note that the periodic forcing in (3) is represented in the jumps \( \Delta_s \theta_b \) and \( \Delta_s q_b \) that depend directly on the diurnal perturbations of the sea surface temperature \( \hat{\theta}_s(x_g) \) and the saturation moisture \( \hat{q}_s(x_g) \). These perturbations are plotted on the bottom right corner of Fig. 2.

As mentioned in Part I, it is not possible to write (3) as an ODE system in explicit form. Thus, a second order accurate one step finite difference method, described in the Appendix of Part I is used to solve these equations. The stability of the resulting periodic solutions

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3. Model results and comparison to observations: The land case

a. The typical stable one-day periodic solution

As is the case of the ocean in Part I, our search for stable periodic solutions is guided by linear stability analysis of the homogeneous RCE steady-state. The results of linear stability analysis are similar to what is presented in Part I and are therefore omitted here. As already pointed out above, while the direct application of Floquet theory to the WK equations is not feasible, nonlinear stability of periodic solutions can be inferred by direct numerical evolution of the system with a small random perturbation added at initial time. Nevertheless, as already pointed out in Part I, the smallness of the velocity components, due to their weak coupling to the convective heating in this climatological setting where wave activity is ignored, can be exploited to reduce the system in (3) by means of asymptotic expansion, into a system that is amenable to an explicit formulation \( y' = f(x, y) \) for which Floquet theory is applicable. All solutions presented in this paper are nonlinearly stable, even though, in some extreme parameter regimes, unstable solutions do exist. They lead to multiday-periodic and chaotic solutions with similar structure as those described in Part I.

A typical stable periodic solution associated with the diurnal cycle forcing over land is plotted in Fig. 2, for the case of a relatively moist RCE background state with \( \bar{\theta}_{eb} - \bar{\theta}_{em} = 6.5 \).
K and a 7.5 K temperature inversion capping the boundary layer. The rest of the model parameters assume the standard values reported in Table 2. The response of the boundary layer potential temperature and moisture are shown in the bottom right corner of Fig. 2. Notice that while $\theta_b$ is more or less in phase with the imposed diurnal heating, which peaks at 12:00 pm, the boundary layer moisture perturbation $q_b$ has a significant lag of about two hours, consistent with the $\theta_e b$ fluctuations seen in Part I for the ocean case. As expected, the velocity components of the periodic solution, shown on the top left panels of Fig. 2, are extremely weak.

The most striking feature of the solution in Fig. 2 is a prominent afternoon peak exhibited by the precipitation plot shown on the bottom left panel, consistent with the deep convective heating, $H_d$, curve on the third row-second column panel. It is important to recall here that we set $f_c = 0$ for the land case and therefore congestus heating does not contribute to the precipitation. In fact, the lack of precipitation from congestus clouds permits a rapid buildup of moisture (see third panel of left column of Fig. 2) in early afternoon which precedes the deep convective maximum. The afternoon deep convective precipitation maximum is consistent with the observed diurnal cycle of precipitation over the Amazon forest (Machado et al. 2004), where the bulk of precipitation begins in the mid-afternoon and peaks in early evening. It should be noted that the structure of diurnal precipitation over the tropical continents is highly dependent on the geographical location, terrain and vegetation type as well as the season. The simplicity of our model may not warrant a detailed comparison with observations, though its qualitative behavior is consistent with land precipitation patterns in the tropics.
b. The physical mechanism of diurnal cycle of precipitation over land

The underlying physical mechanism and dynamical behavior associated with the solutions in Fig. 2 is explained here in terms of the interactions of the three cloud types with the periodically forced boundary layer dynamics. It is evident from the plots in Fig. 2 that the sudden acceleration of the morning deep convection is driven by the rise of $\theta_e$ in the ABL, i.e, CAPE at sunrise. However, this morning deep convection is followed by a rapid warming and drying of the middle troposphere which disfavors deep convection. The warming is due to both deep-convective heating and mixing of ABL sensible heat due to detrainment of shallow cumulus clouds, through the entrainment parameter ($\alpha_E = 1/3$).

The dry and warm environment of the early afternoon combined with the abundance of low level CAPE ($Q_c$) favors congestus heating that slightly warms the lower troposphere and cools the upper troposphere, thus delaying the re-initiation of deep convection. Meanwhile, the mid-troposphere moisture builds up due to shallow cumulus entrainment and detrainment fluxes. The afternoon congestus also can moisten the middle atmosphere through low-level moisture convergence (KM06, KM08) but it is unlikely that this is the main mechanism at work here since the velocity components are very weak. Shortly after 3 PM, the atmosphere reaches a relative humidity level that allows for deep convection. The massive amount of accumulation of CAPE is then discharged quickly in the moist-preconditioned troposphere and results in the explosive afternoon deep convection episode, right before sunset.

After sunset, the tropospheric moisture recovers quickly. However, because of the relative increase in precipitation, the ABL moisture starts dropping, below its equilibrium value shortly after 6 pm due to downdrafts. At this time, the troposphere nearly reaches a ra-
diative convective equilibrium (RCE) state, characterized by a moderate precipitation rate of roughly 1.5 K/day (that roughly balances the imposed radiative cooling) which plateaus between roughly 8:00 pm and 6:00 am. Note a significant decrease of potential temperature $\theta_1$ during the evening between roughly 6:00 pm and 8:00 pm due to the imposed radiative cooling of 1 K/day. The sudden rise in CAPE in the morning combined with the relatively cold troposphere allows for deep convection to rise beyond its overnight equilibrium level at sunrise and closes the cycle.

In summary, the diurnal cycle of precipitation over land can be divided into a cycle of five phases somewhat similar to the Ocean case reported in Part I though with very different physics and different timing: (1) A persistent phase of overnight RCE level precipitation where convective heating balances the imposed long-wave radiation which prevails roughly between 8:00 pm and 6:00 am is followed by (2) a CAPE build up and tropospheric warming phase associated with the early morning warming of the boundary layer which expands between 6:00 am and roughly 10:00 am characterized by a significant rise in precipitation that accelerates the drying and warming of the middle troposphere. (3) A rapid and intense (re)moistening phase of the relatively warm troposphere to hold significant amounts of vapor starts at 10:00 am. This is accompanied by congestus heating that would normally favor further moisture storage. (4) An explosive precipitation peak phase starts at 4:00 pm. It quickly dries the troposphere and consumes CAPE by warming the troposphere to die shortly before 6:00 pm. This phase is followed by (5) a second (re)moistening episode that establishes the moisture level to sustain the overnight precipitation from phase (1).

As demonstrated in the next section through a series of sensitivity tests, the main physical mechanisms that control this five phase cycle of precipitation over land are due to the
combination of four main processes that characterize the present model setup: (1) a strong temperature inversion on the top of the ABL of about 7.5 K; (2) shallow cumulus entrainment and downdraft fluxes of sensible heat corresponding to $\alpha_E = \beta_D = 1/3$; (3) a strong Bowen ratio of about $\beta = 0.4$; (4) non-precipitating congestus heating that allows a rapid and intense moistening during midday and early afternoon ($\xi_c = 0$ for the standard parameters in Table 2).

4. Sensitivity to parameters and comparison to the ocean case

   a. Moist and dry RCE states, inversion and entrainment

   The prototypical solution associated with the standard parameters in Table 2 presented above corresponds to a mildly moist RCE atmosphere $\bar{\theta}_{eb} - \bar{\theta}_{em} = 6.5$ K which, in terms of the switch function $\Lambda$, corresponds to $\bar{\theta}_{eb} - \bar{\theta}_{em} = 14$ K in the ocean setup in Part I. Recall that for the ocean case in Part I, dry tropospheric RCEs are characterized by a strong secondary afternoon congestus precipitation peak for $\bar{\theta}_{eb} - \bar{\theta}_{em} \geq 16$ K. However, this is not the mechanism at work in the development of a strong afternoon precipitation peak here, since congestus clouds do not precipitate in the land setup. However, the magnitude of the afternoon precipitation does increase with a drier RCE state, consistent with the ocean case.

   The effect of the variation of the background moisture on the diurnal cycle of precipitation is depicted at the top of Fig. 3, which compares the standard case to a moister and a drier RCE corresponding to $\bar{\theta}_{eb} - \bar{\theta}_{em} = 4.5$ and 8.5 K, respectively. Similar to the ocean case,
the moist RCE case allows for strong deep convection in the morning, which uses up most of the moisture. As a consequence, the secondary afternoon precipitation peak is weaker in magnitude despite the larger daytime CAPE magnitude. On the other hand, a drier RCE atmosphere is not favorable to strong morning convection due to low relative humidity. Instead, congestus clouds (favored by the nonlinear moisture switch) use up low level CAPE and allow for upper troposphere cooling while the abundant ABL latent heat flux moistens the troposphere in the early afternoon through entrainment mixing associated with shallow cumulus clouds. Since, the troposphere is drier it takes longer to moisten resulting in a delay in the afternoon precipitation peak.

At this point one can be misled to believe that the afternoon deep convection results purely from a drier atmospheric profile setting at RCE and a lack of congestus precipitation invoked in the model by setting $f_c = 0$. However, these two mechanisms by themselves result only in a small afternoon deep convective peak. Other changes in parameters, namely a strong temperature inversion at the top of the ABL, a large Bowen ratio, and non-zero entrainment and downdrafts fluxes of sensible heat ($\alpha_E \neq 0, \beta_D \neq 0$) are all necessary to produce the observed strong afternoon precipitation, as demonstrated below.

The sensitivity of the precipitation cycle to the strength of the temperature inversion is presented in the middle of Fig. 3. Strong inversions are frequently encountered above the ABL of the Amazon forest (Fu et al. 1999). The inversion is maximum in the morning and acts as a barrier inhibiting deep convection. In observations (Fu et al. 1999), it gets eroded during the day due to surface latent and sensible heat fluxes. Consistently, this is roughly what is seen on the top right panel of Fig. 2 for the multi-cloud model.

We recall that the WK model is altered to include the redistribution of the entrainment
and downdraft fluxes between the free tropospheric moisture and the potential temperature to allow penetration of sensible heat fluctuation through the top of the boundary layer interface. For the ocean case, discussed in Part I, these effects are negligible due to the weak Bowen ratio and weak temperature inversion. The inclusion of strong inversion here allows the boundary layer potential temperature fluctuation to impact significantly the temperature of the free troposphere through first baroclinic temperature mode. Thus ABL sensible heat warms the deep atmosphere during the day and inhibits deep convection.

Sensitivity tests of the diurnal cycle of precipitation due to changes in the entrainment and downdraft parameters (\(\alpha_E\) and \(\beta_D\)) are depicted on the bottom panel of Fig. 3. No exchange of sensible heat at the top of the ABL (\(\alpha_E = \alpha_D = 0\)) results in a colder mid-troposphere (i.e., higher relative humidity with a small amount of moisture) and thus a stronger and more efficient morning deep convection. This scenario still creates an afternoon peak but it is comparable in amplitude to the morning convection due to the smaller moisture content. High values of the entrainment and downdraft parameter (\(\alpha_E = \beta_D = 2/3\)) allow faster warming of atmosphere by ABL sensible heat perturbation (which precedes the moisture fluctuation) and permits higher moisture loading and results in a much stronger afternoon precipitation peak.

Note that changes in RCE dryness, inversion strength, and entrainment and downdraft parameters, are accompanied with an adjustment of the ABL cumulus updraft parameter \(\alpha_m\) and the surface flux \(\Delta_s \tilde{\theta}\) in order to preserve comparable ABL cooling rate \(Q_{RB}\) and evaporation time scale \(\tau_e\). Note that while the magnitude of the RCE precipitation changes, the above analysis is based on the relative sizes of morning and afternoon precipitation peaks. Note in particular that a moister troposphere results in a higher overnight RCE precipitation
rate that in turn affects the strength of the whole solution and the afternoon precipitation peak in particular.

b. *Bowen ratio and dry buoyancy frequency*

A Bowen ratio of $\beta = 0.4$ is used here as a standard value to represent a typical case of precipitation over tropical forests (Fu et al. 1999). The sensitivity tests due to changes in the Bowen ration are depicted on top of Fig. 4. Note that the case $\beta = 0$ is similar to the ocean scenario reported in Part I, characterized by a dominant morning precipitation peak, despite the fact that both the strong inversion and sensible heat entrainment and downdrafts are activated. The ABL potential temperature variation is very small and therefore convection is the sole source of heating of the deep troposphere. The morning deep convection dries the troposphere without a significant warming from sensible heat that would allow a rapid storage of moisture. Thus, the subsequent afternoon deep convection peak is weaker. Meanwhile, the higher Bowen ratio ($\beta = 0.8$) allows for faster morning warming of the troposphere. Therefore, morning deep convection stops earlier due to additional heating by sensible heat entrainment which at the same time allows for significant storage of moisture around noon and early afternoon. Therefore, the afternoon peak is larger in magnitude and is slightly delayed since it takes longer for the warmer troposphere to cool down by the fixed radiation. The higher precipitation peaks, occurring at higher Bowen ratios, are due to larger total perturbations of surface fluxes, which result in higher CAPE anomalies.

The bottom panel of Fig. 4 displays sensitivity tests to variations in the dry-buoyancy parameter $a_0$, used in the closure of the deep convection potential, $Q_d$ (see Table 1), so that
a colder middle troposphere facilitates deep convection by allowing saturation at weaker mixing ratios (KM06a). Due to the emphasis on the penetrative sensible heat flux in this paper, we choose a higher value of convective buoyancy parameter \( (a_0 = 14) \) as the standard case, whereas it is set to \( a_0 = 7 \) for the ocean study in Part I.

From the bottom panel of Fig. 4, we see that as expected higher \( a_0 \) values result in higher afternoon and lower morning precipitation peaks and vice-versa for lower \( a_0 \). The higher \( a_0 \) value makes deep convection more sensitive to temperature variations; thus the rise in the ABL potential temperature perturbation successfully reduces the amplitude of the morning deep convective precipitation and allows storage of greater amounts of moisture to be available for the afternoon precipitation, as already anticipated.

c. Sensitivity to stratiform and congestus rain fractions

Recall that in KM08 the rain fractions due stratiform and congestus cloud types are given, as averages relative to variations in the moisture switch function \( \Lambda \), by

\[
    f_s = \xi_s \alpha_s (1 + \xi_s \alpha_s + \xi_c \alpha_c)^{-1}
\]

and

\[
    f_c = \xi_c \alpha_c (1 + \xi_s \alpha_s + \xi_c \alpha_c)^{-1}.
\]

The standard values used in this paper \( f_s = 0.4, f_c = 0 \) are chosen in accordance with the observational data according to which congestus clouds contribute very little to the total precipitation over land, while stratiform and deep convective precipitation contribute about 40% and 60%, respectively (Schumacher and Houze 2003; Takayabu et al. 2009). Stratiform rain fraction is observed to vary significantly with the geographic location all over the tropics. However, land regions tend to have lower stratiform rain fractions and higher deep convective activity. This is particularly the case over central Africa where stratiform rain accounts for about 20% to 30% of the total precipitation.
However, the central Amazon region has an average stratiform precipitation of about 30% to 40% (Casey et al. 2007), which motivated our standard value $f_s = 0.4$.

Sensitivity of the model to the stratiform rain fraction $f_s$ is presented at the top panel of Fig. 5. We note that with lower stratiform rain fraction the afternoon peak in total precipitation is larger. Recall that in the multicloud model, the stratiform clouds lag deep convection, which suggests the following interpretation. A smaller fraction of stratiform precipitation allows for a higher moisture build up during the non-precipitating congestus stage and results in a larger afternoon deep convection peak. It is interesting to note that similar to the ocean case the amplitude of precipitation in the morning is not very sensitive to the variation of the stratiform rain fraction. We conclude by noting that lower stratiform fractions are perhaps more appropriate for the land regime since they tend to produce higher afternoon precipitation peaks.

Similarly, sensitivity tests to variations in the congestus rain fraction $f_c$ are depicted on the bottom panel of Fig. 5. As mentioned above non-precipitating congestus is one of the key parameters that directly control the afternoon precipitation peak associated with the land regime. A small congestus precipitation fraction allows the afternoon congestus clouds to rain from noon to 3:30 pm without exhausting all of the moisture stored during the morning which nonetheless permits a second-afternoon peak in deep convective precipitation that is roughly similar to the standard case corresponding to $f_c = 0$ but somewhat smaller in magnitude. The high congestus fraction $f_c = 0.2$, on the other hand, induces a significant drying of the troposphere by congestus rain and results in a weak (non-existing in this extreme case) afternoon deep convection. Even though we have a high afternoon precipitation peak due to the high congestus rain fraction in the continental regime, this solution is unphysical since,
as mentioned above, the bulk of the tropical land precipitation comes from deep convection and the trailing stratiform clouds (Casey et al. 2007; Takayabu et al. 2009).

5. Concluding summary and discussion

The multicloud model (Khouider and Majda 2006, 2008b) with full bulk boundary layer dynamics (Waite and Khouider 2009) is used here to study the diurnal cycle of tropical precipitation over land. The case of precipitation over the ocean is presented in Part I (Frenkel et al. 2010). The main model consists of the two first vertical baroclinic modes of vertical structure forced by heating profiles based on the three cloud types that characterize organized tropical convection: cumulus congestus, deep convection, and trailing stratiform cloud decks, coupled to a thin boundary layer that responds to solar surface heating and downdrafts. This particular iteration of the model, introduced in Waite and Khouider (2009), incorporates the bulk-boundary layer equations of Stevens (2006) to facilitate the treatment of sensible and latent heat fluxes separately and includes entrainment and detrainment fluxes associated with shallow cumulus. Additional features of the present setup to accommodate tropical convection over land include a strong surface sensible heat flux (a large Bowen ratio), a strong temperature inversion, and non zero entrainment and downdraft fluxes of sensible heat. The main result here is a stable periodic solution characterized by a large afternoon precipitation peak resembling observations of the diurnal cycle of precipitation over tropical rain forests, e.g. Amazonia (Machado et al. 2004).

The equations of motion along the equator—without rotation and without meridional dependence, forced by latent and sensible heat surface fluxes, are written in a moving frame
circling the globe at a constant speed of one rotation per day following the local solar time (LST), to facilitate the analysis. Time independent solutions depicting the diurnal variations of the climatology where the effects of waves and other non-homogeneities are filtered out are sought in the form of stable periodic solution of an ODE system with periodic coefficients.

The present study highlights the importance of capturing interactions of the free troposphere with the boundary layer which is directly forced by the diurnal cycle of solar heating in terms of both sensible and latent heat fluxes. Since the model allows for efficient coupling between the boundary layer and free tropospheric potential temperature, the surface sensible heat flux warms the atmosphere in the morning lowering the relative humidity and impeding deep convection. Boundary layer moisture, which peaks in the afternoon, combined with detrainment of shallow cumulus and non-precipitating cumulus congestus clouds rapidly moistens the previously warmed troposphere. This then results in a pronounced afternoon precipitation maximum. According to the present model results, the detailed diurnal cycle of precipitation over land is divided into a cycle of five phases: (1) A persistent phase of overnight RCE level precipitation where convective heating balances the imposed long-wave radiations prevailing roughly between 8:00 pm and 6:00 am is followed by (2) a CAPE build up phase associated with the early morning warming of the boundary layer, which extends between 6:00 am and roughly 10:00 am is accompanied by a significant rise of precipitation that accelerates the drying and contributes to the warming of the middle troposphere; these effects in addition to the indirect warming due to mixing from shallow cumulus entrainment overcome the imposed radiative cooling. (3) A rapid and intense (re)moistening phase then starts at 10:00 am and is followed by congestus heating. (4) An explosive precipitation peak phase starts at 4:00 pm due to the large amount of CAPE and moisture stored in the previ-
ously warmed atmosphere. It quickly dries the troposphere and consumes CAPE by further warming the troposphere and dies slightly before 6:00 pm. The latter is followed by (5) a second (re)moistening episode that (re)establishes the moisture level to sustain the overnight precipitation of phase (1) and closes the cycle.

Sensitivity tests which determine the main physical parameters, that are crucial for explaining the phase shift in the precipitation maximum, from morning to afternoon, when compared to the ocean case reported in Part I, are conducted and presented in section 4. The results in section 4 demonstrate that the manifestation of an afternoon peak in deep convection is controlled by the combined effects of (1) a large Bowen ratio (a relatively large surface flux of sensible heat) of about $\beta = 0.4$, (2) a strong temperature inversion at RCE of about -7.5 K, (3) significant mixing of sensible heat at the top of the ABL due entrainment and downdraft fluxes ($\alpha_E = \beta_D = 1/3$), and (4) weakly or non-precipitating congestus heating ($f_c = 0$) to allow significant moisture build up at noon and during early afternoon.

Although simple, the present model allows us to derive a physically sound five phase cycle of tropical precipitation over land which is characterized by a strong afternoon precipitation maximum consistent with observations (Machado et al. 2004) and suggest a minimal set of physical parameters that are crucial for its occurrence.

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<th>Name</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum, 0th mode</td>
<td>$\frac{\partial \bar{u}}{\partial t} = -\partial_x p_0 - \partial_x \bar{u}(2\bar{u} - u_b) + \frac{E_u}{HT} \Delta_t u$</td>
</tr>
<tr>
<td>Momentum, jth mode, $j = 1, 2$</td>
<td>$\frac{\partial u_j}{\partial t} - \partial_x \theta_j + \partial_x (\bar{u} u_j) = \frac{\sqrt{2}}{\tau_R} \delta_b \Delta_t u - \frac{1}{\tau_R} u_j$</td>
</tr>
<tr>
<td>Potential temperature, 1st mode</td>
<td>$\frac{\partial \theta_1}{\partial t} - \partial_x u_1 + \bar{u} \partial_x \theta_1 + \sqrt{2} \partial_x \bar{u} = H_d + \xi_s H_s + \xi_c H_c + S_1 + \frac{\pi}{2\sqrt{2}} \left( \frac{\alpha E}{HT} \right) \Delta_t \theta + \beta_D \left( \frac{M_d}{HT} + \partial_x \bar{u} \right) \Delta_m \theta$</td>
</tr>
<tr>
<td>Potential temperature, 2nd mode</td>
<td>$\frac{\partial \theta_2}{\partial t} - \frac{1}{4} \partial_x u_1 + \bar{u} \partial_x \theta_2 + \frac{\sqrt{2}}{4} \partial_x \bar{u} = H_c - H_s + S_2$</td>
</tr>
<tr>
<td>Radiative cooling</td>
<td>$S_i = -Q_{R,i}^0 - \tau_D^{-1} \theta_i$</td>
</tr>
<tr>
<td>Free tropospheric moisture</td>
<td>$\frac{\partial q}{\partial t} + \bar{u} \partial_x q + \partial_x [(u_1 + \delta u_2)q + (u_1 + \bar{\lambda} u_2)\bar{Q} - \bar{u} \bar{Q}_0] = -\frac{2\sqrt{2}}{\pi} (H_d + \xi_s H_s + \xi_c H_c) + \frac{E}{HT} \Delta_t q + \left( \frac{M_d}{HT} + \partial_x \bar{u} \right) \Delta_m q + (1 - \alpha_E) \frac{E}{HT} \Delta_t \theta + (1 - \beta_D) \left( \frac{M_d}{HT} + \partial_x \bar{u} \right) \Delta_m \theta$</td>
</tr>
<tr>
<td>Boundary layer potential temperature</td>
<td>$\frac{\partial \theta_b}{\partial t} + u_b \partial_x \theta_b = -\frac{E}{h_b} \Delta_t \theta - \frac{M_d}{h_b} \Delta_m \theta + \frac{1}{\tau_e} \Delta_s \theta - Q_{Rb}$</td>
</tr>
<tr>
<td>Boundary layer moisture</td>
<td>$\frac{\partial q_b}{\partial t} + u_b \partial_x q_b = -\frac{E}{h_b} \Delta_t q - \frac{M_d}{h_b} \Delta_m q + \frac{1}{\tau_e} \Delta_s q$</td>
</tr>
<tr>
<td>Boundary layer velocity</td>
<td>$\frac{\partial u_b}{\partial t} + \bar{u} \partial_x \bar{u} = -\partial_x p_0 - \partial_x \bar{u}(\bar{u} - u_b) + \frac{E_u}{HT} \Delta_t u$</td>
</tr>
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</table>
### Table 1 (Continued)

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convective downdraft</td>
<td>$D_c = m_0 [1 + \mu (H_s - H_c) / Q_{R;1}]^{-1} \Delta_m \theta_c$</td>
</tr>
<tr>
<td>Total downdraft</td>
<td>$M_d = (D_c + h_b \partial_x u_b)^+$</td>
</tr>
<tr>
<td>Scalar entrainment velocity</td>
<td>$E = (M_u - M_d + h_b \partial_x u_b)^+$</td>
</tr>
<tr>
<td>Momentum entrainment velocity</td>
<td>$E_u = (\frac{h_b}{\tau_T} + h_b \partial_x u_b)^+$</td>
</tr>
<tr>
<td>Stratiform heating</td>
<td>$\frac{\partial H_s}{\partial t} = \frac{1}{\tau_s} (\alpha_s H_d - H_s)$</td>
</tr>
<tr>
<td>Congestus heating</td>
<td>$\frac{\partial H_c}{\partial t} = \frac{1}{\tau_c} (\alpha c \Lambda Q_c^+ - H_c)$</td>
</tr>
<tr>
<td>Deep convection</td>
<td>$H_d = (1 - \Lambda)Q_d^+$</td>
</tr>
<tr>
<td>Maximum energy available for deep convection</td>
<td>$Q_d = \bar{Q} + \tau_{conv}^{-1} [a_1 \theta_{eb} + a_2 q - a_0(\theta_1 + \gamma_2 \theta_2)]^+$</td>
</tr>
<tr>
<td>Maximum energy available for congestus convection</td>
<td>$Q_c = \bar{Q} + \tau_{conv}^{-1} [\theta_{eb} - a'_0(\theta_1 + \gamma'_2 \theta_2)]^+$</td>
</tr>
<tr>
<td>Jump between the surface and bulk ABL value of $\phi$</td>
<td>$\Delta_s \phi = \phi_s - \phi_b$</td>
</tr>
<tr>
<td>Jump between bulk ABL and top of ABL value of $\phi$</td>
<td>$\Delta_t \phi = \phi_b - \phi_t$</td>
</tr>
<tr>
<td>Jump between bulk ABL and middle troposphere value of $\phi$</td>
<td>$\Delta_m \phi = \phi_b - \phi_m$</td>
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</tbody>
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Table 2. Constants and parameters for the case of the land regime using the WK model.

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_b$ / $H_T$ / $\delta$</td>
<td>500 m / 16 km / 0.03125</td>
<td>ABL depth / Free troposphere depth / ratio of $h_b$ to $H_T$</td>
</tr>
<tr>
<td>$Q_{R1}$</td>
<td>1 K/day</td>
<td>First baroclinic radiative cooling rate</td>
</tr>
<tr>
<td>$Q_{R2}$</td>
<td>Determined at RCE</td>
<td>Second baroclinic radiative cooling rate</td>
</tr>
<tr>
<td>$Q_{Rb}$</td>
<td>Determined by RCE</td>
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</tr>
<tr>
<td>$\xi_s$ / $\xi_c$</td>
<td>$\frac{8}{3}$ / 0</td>
<td>Stratiform / Congestus contribution to first baroclinic mode</td>
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<tr>
<td>$\tilde{Q}$ / $\tilde{Q}_0$</td>
<td>0.9 / 6.5</td>
<td>Background moisture stratification / contribution to barotropic vertical moisture advection</td>
</tr>
<tr>
<td>$\tilde{\lambda}$ / $\tilde{\alpha}$</td>
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<td>$m_0$</td>
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<td>$\tau_R$ / $\tau_D$</td>
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<td>Determined at RCE</td>
<td>Bulk convective heating at RCE</td>
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<tr>
<td>$a_1$ / $a_2$</td>
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<td>Relative contribution of $\theta_{eb}$ / $q$ to deep convection</td>
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<td>$a_0$ / $a_0'$</td>
<td>7 / 1.5</td>
<td>Dry convective buoyancy freq. in deep / congestus eq.</td>
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<tr>
<td>$\gamma_2$ / $\gamma_2'$</td>
<td>0.1 / 2</td>
<td>Relative contribution of $\theta_2$ to deep / congestus heating</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.1</td>
<td>Relative contribution of $\theta_2$ to $\theta_{em}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>2</td>
<td>Ratio of $q_l$ to $q$</td>
</tr>
<tr>
<td>$C_d$</td>
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<td>Surface drag coefficient</td>
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<tr>
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<td>Strength of turbulent fluctuations</td>
</tr>
<tr>
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2 Stable periodic response of the WK model to a diurnal cycle forcing in the land regime for the case of a moist background corresponding to $\bar{\theta}_{eb} - \bar{\theta}_{em} = 6.5$ K (comparable to $\bar{\theta}_{eb} - \bar{\theta}_{em} = 14$ K for original set up–without an inversion). The rest of the parameters are fixed to their standard values in Table 2. 36

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