PARTIAL DIFFERENTIAL EQUATIONS, MATH-GA.2500

SPRING 2020

Tues 9:00–10:50, WWH 517

Professor Robert Kohn
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Spring 2020 office hrs: Mon 5-6 and Thurs 4-5

Revised 4/12/2020 concerning the Final Exam, which has been changed to a take-home exam on **Tues May 19**.

Prerequisites: MATH-GA 2490 (Introduction to Partial Differential Equations) and MATH-GA 2430 (Real Variables), or similar background. *Masters students should consult the instructor before registering for this class.*

Description: Undergraduate and MS-level classes in PDE usually emphasize examples, involving solutions that are more or less explicit. This course does the opposite: it emphasizes more general methods, applicable to broad classes of PDE's. Topics to be covered include: tools from analysis (Fourier transform, distributions, and Sobolev spaces, including embedding and trace theorems); linear elliptic pde (weak solutions, regularity, Fredholm alternative, symmetry and self-adjointness, completeness of eigenfunctions; maximum principles and Perron's method; boundary integral methods); selected methods for solving nonlinear elliptic pde (fixed point theorems, variational methods); parabolic and hyperbolic pde (energy methods, semigroup methods, steepest-descent pde's); viscosity solutions of first-order equations.

Texts: We will draw mainly on

(1) L.C. Evans, *Partial Differential Equations*, American Mathematical Society, 2nd edition (2010)

(2) M. Renardy and R. Rogers, *An Introduction to Partial Differential Equations*, Springer-Verlag (2004)

(3) G. Folland, *Introduction to Partial Differential Equations*, Princeton University Press, 2nd edition (1995).

I strongly suggest you buy or borrow a copy of Evans. (The first edition will be sufficient for most purposes. The 2nd edition has additional exercises and some additional sections; if we draw on the sections that aren't in the first edition it will only be in one or two lectures toward the end of the semester).

Renardy & Rogers is available through Springerlink: from an NYU computer (or using Bobcat) you can download a pdf for free, and you can buy a paperback version for about $25. Evans and Renardy/Rogers complement each other nicely, in the sense that while they are both at about the same level, they emphasize different things – for example, Evans downplays the theory of distributions while it is prominent in Renardy/Rogers; and Evans

downplays Fourier-transform-based methods while this too is prominent in Renardy/Rogers.

Folland is like Renardy/Rogers in some of his choices (for example, he too emphasizes Fourier-based methods) but his viewpoint is less functional-analytic. This may make Folland easier to read than Renardy/Rogers, for those topics that are treated in both books. Folland is a very good book, well worth owning, but we will do relatively few things that are only there; so if to save money you want to get just two books, get Evans and Renardy/Rogers and xerox what you need from a friend's copy of Folland (or the one on reserve in the CIMS library).

All three of these books were written to support a graduate class that starts from the beginning; so each begins with material corresponding roughly to MATH-GA.2490. (Don't be frightened: the initial sections of these books surely contain plenty that you didn't learn in whatever introductory course you took!)

There are also plenty of other good books on PDE. I will distribute a separate list of some that you might find useful.

Course requirements: There will be about 6 homework assignments and a final exam. The HW and final will count equally toward the grade (i.e. grade = 50%HW + 50%Final Exam). **Revised April 2020:** *Since we are online-only through the end of the semester, we will have a "take-home" final exam. Also, while my original plan for our final exam was to hold it on May 12 ("Reading Day"), my revised plan is to place it one week later, on Tuesday May 19 (the Tuesday that's part of NYU's Final Exam period). The Final will be designed to take you a couple of hours; but since students are now in various time zones, the exam will be released at 9am Tuesday morning 5/19 (NY time), and you'll need to upload your solution by 9am Wednesday morning 5/20 (NY time). You will be permitted to use your notes, textbooks, other books, internet resources, etc; however you will be expected not to consult with other people (classmates or otherwise). If the shift to May 19 poses a problem for you, please send me an email ASAP explaining your situation.*

Collaboration on homework: Some HW problems may be routine, but some will be not-so-routine. Collaboration is encouraged, and looking in books is OK too. However each student must write up his/her own solutions (this is an important part of the learning process). If you work closely with someone else or take a solution directly from a book or some other source, please *identify your collaborators and/or sources.* Direct copying of another student's solutions is *not* permitted – both because it is considered cheating, and because it is defeats the entire purpose of the homework (which is to gain practice and familiarity with new concepts and techniques).

Lectures vs reading: You need to know more than can possibly be covered in one semester's worth of lectures. Moreover, well-organized arguments with full details are important, but they aren't necessarily the best use of our lecture time. I will often discuss the main ideas of an argument, or a special case of a result, referring to one of our books for a more complete and detailed discussion. *Read the suggested material* and make sure

you understand it. Your HW solutions should be complete (like what might be found in a book) rather than sketchy (as I might do in a lecture).

HW extensions and makeup exams: Each HW will have a definite due date. I will entertain requests for brief extensions, however late HW will not be accepted once a solution sheet has been distributed. Requests to take a makeup final will be considered only when there is a legitimate reason (such as documented illness). If the reason for requesting a makeup exam is known in advance, permission to take a makeup must be requested *before* the exam date.

Academic integrity: Plagiarism and cheating will not be tolerated. NYU's Graduate School of Arts and Sciences has policies in this area, and they will be followed.