Partial Differential Equations for Finance
G63.2706, Spring 1999
Wednesdays 7-8:50
406 Main

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Special Dates: First lecture Jan. 20. No lecture March 10 (out of town) and March 17 (spring break). Last lecture April 28. Final exam: May 5.

Content: An introduction to those aspects of partial differential equations most relevant to finance. First order equations: dynamic programming, Hamilton-Jacobi equations, viscosity solutions. Parabolic equations: fundamental solution, boundary value problems, maximum principle, variational inequalities. Diffusion processes: Brownian motion, probabilistic interpretation of the heat equation, Feynman-Kac formula, Cameron-Martin-Girsanov formula. Stochastic games: dynamic programming with uncertainty, stopping times, free boundary problems. Various applications.

Prerequisites: Calculus and discrete probability. No specific knowledge of partial differential equations or stochastic processes will be assumed. However students will need considerable scientific maturity, at a level typically obtained through a firm undergraduate background in economics or the physical sciences.

Course requirements: There will be several homework sets, one every couple of weeks, probably 5 or 6 in all. Collaboration on homework is encouraged (homeworks are not exams) but registered students must write up and turn in their solutions individually. There will be one in-class final exam.

Lecture notes: Lecture notes and homework sets will be handed out, and also posted on my website as they become available. (Go to the CIMS home page www.cims.nyu.edu and select faculty to find me.)

Books: There is no textbook for this course. Following is a list of books that some students may find helpful. Most of them are on reserve in the CIMS library. The first five have been ordered by the NYU bookstore. *Read my comments before you buy any of these books.* I will sometimes place xeroxed articles or sections from books on reserve. They'll be in the "Green box" with my name on it, which you can request from the CIMS library staff.

- I. Capuzzo Dolcetta and P.-L. Lions, editors, *Viscosity solutions and applications*, Lecture Notes in Mathematics No. 1660, Springer-Verlag, 1997, approx $50. A mathematically sophisticated book, far beyond the level of this course. May be of interest to students who already know a lot of PDE. The chapters relevant to this course are the ones by Crandall, Bardi, and Soner.
- P. Wilmott, J. Dewynne, and S. Howison, *The mathematics of financial derivatives: a student introduction*, Cambridge University Press, 1995, approx $30. A treatment of the Black-Scholes theory written from the perspective of PDE. Students interested in PDE and finance will likely find this good value for money. But only the brief Chapter 4 ("Partial Differential Equations") is really in the spirit of this course.

- M. Baxter and A. Rennie, *Financial calculus: an introduction to derivative pricing*, Cambridge University Press, 1996, approx $40. An extremely readable, relatively nontechnical treatment of derivative pricing theory from the viewpoint of martingales and risk-neutral measures. Students interested in stochastic PDE applied to finance will likely find this good value for money (and a nice complement to Wilmott, Dewynne, & Howison). Our treatment of stochastic differential equations, martingales, and the like will draw on this.

- J. Ingersoll, *Theory of financial decisionmaking*, Rowman and Littlefield, 1987, approx $65. A financially sophisticated book, far beyond the actual level of this course. May be useful to students who already know a lot of economics.

- D. Lamberton and B. Lapeyre, *Introduction to stochastic calculus applied to finance*, Chapman and Hall, 1996, approx $65. More difficult than Wilmott-Dewynne-Howison and Baxter-Rennie, hence *a fortiori* more sophisticated than the level of this course. There's a lot of derivative pricing here, not much PDE or basic stochastic processes.

- L. C. Evans, *Partial differential equations*, American Math Society, 1998, approx $75. The best modern basic-level introduction to PDE I know. We'll draw from this book for deterministic control and parabolic PDE. (Not yet received by the CIMS library – will be on reserve once they get it. AMS phone number for orders: 1-800-321-4267).

- F. John, *Partial differential equations*, 4th edition, Springer-Verlag. This was the best basic-level introduction before Evans' book appeared. Good treatment of parabolic PDE, but no control theory or dynamic programming.

- Ludwig Arnold, *Stochastic differential equations*, Krieger. A good introduction to stochastic differential equations and their applications. Alas, it's out of print.

- S. Neftci, *An introduction to the mathematics of financial derivatives*, Academic Press, 1996, approx $40. This book is at the same level as Wilmott-Dewynne-Howison and Baxter-Rennie: a nontechnical introduction to PDE's and stochastic processes applied to pricing derivative securities.

- W. Fleming and R. Rishel, *Deterministic and stochastic optimal control*, Springer-Verlag, 1975. A very good introduction to dynamic programming and optimal control in the continuous-time setting. The examples are chosen mainly from the physical sciences not finance.

- Stanley Pliska, *Introduction to mathematical finance: discrete time models*, Blackwell, 1997, approx $55. There's not a single PDE here, since time is discrete, but it's a good treatment of the key issues in the discrete-time setting.

- Dimitri Bertsekas, *Dynamic programming: deterministic and stochastic models*, Prentice-Hall, 1987. No PDE here either, but a good treatment of discrete-time dynamic programming.

- Paul Wilmott, *Derivatives*, John Wiley and Sons, 1998, approx $80. I just learned about this book, thought you might like to know about it too. It's a much-expanded and updated version of the little Wilmott-Dewynne-Howison book. (Only a few short chapters correlate with this course however.)