PDE I
MATH-GA 2490, Fall 2014
Tuesdays 5:10-7:00pm, WWH 102

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Recent change: added office hours

Instructor: Robert Kohn. Office: 502 WWH. Phone: 8-3217. Email: kohn@cims.nyu.edu. Web: math.nyu.edu/faculty/kohn. Office hours: Mondays and Thursdays 5-6.

Prerequisites: A good knowledge of undergraduate-level linear algebra and ODE; also some exposure to complex variables (can be taken concurrently). This is an introductory course, but it will move quickly and require considerable mathematical maturity. The course is aimed mainly at entering PhD students, but is suitable also for well-prepared MS students.

Course description: A basic introduction to PDEs, designed for a broad range of students whose goals may range from theory to applications. This course emphasizes examples, representation formulas, and properties that can be understood using relatively elementary tools. We will take a broad viewpoint, including how the equations we consider emerge from applications, and how they can be solved numerically. Topics will include: the heat equation; the wave equation; Laplace's equation; conservation laws; and Hamilton-Jacobi equations. Methods introduced through these topics will include: fundamental solutions and Green's functions; energy principles; maximum principles; separation of variables; Duhamel's principle; the method of characteristics; numerical schemes involving finite differences or Galerkin approximation; and many more.

Homeworks, exams, grades: There will be weekly homework sets, an in-class *midterm exam on Tues 10/21* , and a *final exam on Tues 12/16* (in the normal class time and place: 5:10-7pm, WWH 102). The semester grade will be based on the HW (1/3), the midterm (1/3), and the final exam (1/3).

Collaboration on homework is encouraged (homeworks are not exams) but registered students must write up and turn in their solutions individually. If you work with another student, please name him or her on your solution sheet. HW may be turned in late only by securing permission on or before the due date. Doing the HW is important, not only because it counts as part of the grade, but also because if you don't do the HW, you probably won't do well on the exams. Copying solutions from other students (or past solution sheets) is not permitted, and not a good idea: you won't have the intended experience, and won't learn the material well.

The midterm and final exams will be closed-book, but you may bring one sheet of notes (8.5×11, both sides, any font) to the midterm, and two sheets of notes to the final. Requests to take a makeup exam must be made in advance, and will *not* be granted for matters of personal convenience.

Books: I will not follow any single book linearly. But you need to learn much more than I can cover in class in 14 weeks. I strongly recommend that you buy (or borrow) the following books, which correlate well with this class:

- R. Guenther and J. Lee, *Partial Differential Equations of Mathematical Physics and Integral Equations*, Dover, 1996. An excellent text, at the right level. I especially like its discussions of where the PDE's come from. By the way: this book's treatment of boundary integral methods for Laplace's equation is among the best I know. Great value for money – Amazon's price is about $15.

- L.C. Evans, *Partial Differential Equations*, American Mathematical Society, 2nd edition, 2010. The first 250 pages (Chapters 2, 3, and 4) correlate with PDE I. The rest of the book presents more advanced material, and will be useful if you continue to PDE II. Amazon's price: around $80. (The first edition is available used for much less; the 2nd edition has some new material in the later part, but the first few chapters didn't change much except for some additional problems.)

There are, of course, many other PDE books. Here is a list of some you might find useful (all will be on reserve in the CIMS Library):

More basic than this class:

- W. Strauss, *Partial Differential Equations: An Introduction*, John Wiley and Sons, 2007. The best undergraduate-level text I know. Many of the topics we'll discuss are present in Strauss, at least in some measure, with an exposition that may be more accessible, especially if your PDE background is limited. I strongly recommend reading the relevant sections of this book alongside Evans and Guenther & Lee.

- H. Weinberger, *A First Course in Partial Differential Equations, with Complex Variables and Transform Methods*, Dover, 1965. The first half is a lot like Strauss: a discussion of PDE making heavy use of separation of variables, but also emphasizing that there's much more to the theory than that. The second half is a good, application-oriented introduction to complex variables. Feels a little dated by now, but this material hasn't changed since 1965 and you can't beat the price (around $15).

About the same level as this class:

- Q. Han, *A Basic Course in Partial Differential Equations*, American Mathematical Society, 2011. Most of the book focuses on the constant-coefficient linear heat, wave, and Laplace equations, viewed from many viewpoints, and with lots of good problems. There's relatively little about nonlinear PDE, except first-order eqns and the method of characteristics. Overall: an excellent book, narrower than this class but at the same level.

- F. John, *Partial Differential Equations*, 4th edition, Springer-Verlag 1982. A classic text, and still one of the best, though a little more sophisticated than this class. Two drawbacks: spends little time discussing where the PDE's come from, and offers relatively few exercises. *Downloadable for free from within the nyu.edu domain* from the website http://link.springer.com/book/10.1007/978-1-4684-0059-5/page/1

(The same page will also offer you the opportunity to order an inexpensive soft-cover edition.) You can also download it from outside NYU, provided you have an NYU-Home account, by using the NYU proxy server; see
http://cims.nyu.edu/webapps/content/systems/userservices/netaccess/proxy
for information on that.

- P. Garabedian, *Partial Differential Equations*, 2nd revised edn, AMS Chelsea Publishing, 1998. Excellent treatment of many basic topics (eg the heat eqn, the wave eqn, Laplace's eqn, first-order eqns) as well as some less basic topics (eg integral equations, application of PDE to fluid dynamics, finite-difference-based numerical methods). Like John, this text has stood the test of time.

- J. Kevorkian, *Partial Differential Equations: Analytical Solution Techniques*, 2nd edition, Springer-Verlag, 1999. Another old standby. Kevorkian is especially fond of explicit solution formulas (more so than me). Many good problems. *Downloadable for free from within the nyu.edu domain* from the site
http://link.springer.com/book/10.1007/978-1-4757-3266-5/page/1
(or using the NYU proxy server, see above).

- J. Ockendon, S. Howison, A. Lacey, and A. Movchan, *Applied Partial Differential Equations*, Oxford University Press, Revised Edition 2003. Very concrete and practical. Good discussions of the physical or probabilistic motivations for various PDE's. Plenty of problems. But: perhaps this book spends *too* much time discussing the applications – it's a bit like drinking from a firehose.

Books that start at our level, but are mainly more advanced:

- G. Folland, *Introduction to Partial Differential Equations*, 2nd edition, Princeton University Press, 1995. Chapters 2 (The Laplace Operator), 4 (The heat operator), and 5 (The wave operator) are largely at the level of this class. The rest of the book is more advanced (at the level of PDE II). However: Folland is a fine complement to Evans, since it emphasizes Fourier-transform-based methods (which Evans de-emphasizes).

- M. Renardy and R. Rogers, *An Introduction to Partial Differential Equations*, 2nd edition, Springer-Verlag, 2004. This book spends even less time on our material than Evans or Folland, and covers it less systematically; thus it is mainly more advanced, at the level of PDE II. One distinguishing feature is its treatment of evolution equations, which has separate chapters on energy-based and semigroup-based techniques.

For thoughtful reviews by J. David Logan that discuss and compare some of these books, see SIAM Review 42 (2000) 515-522 and SIAM Review 51 (1999) 393-395.

Tentative semester plan:

Lectures 1-3: The heat equation and related topics. Examples of problems leading to the heat equation or closely related equations (probability as well as physics). Solution formulas for the Cauchy problem, half-space problems, and bounded domains.

Backward eqn is ill-posed. Uniqueness via energy argument and via maximum principle. Some basic numerical schemes (finite differences, Galerkin approximation, explicit vs implicit time-stepping). Some nonconstant-coefficient and nonlinear problems that can be done using similar techniques.

Lectures 4-7: Laplace's equation and related topics. Examples of problems leading to Laplace's equation, or closely related equations (including probability and fluid dynamics). Solution formulas using complex variables, the fundamental solution, separation of variables, and the Green's function. Uniqueness via energy argument and via maximum principle. Variational principles. Some basic numerical schemes (finite differences, finite elements, Galerkin approximation). Some non-constant-coefficient and nonlinear problems that can be done using similar techniques.

No class Oct 14, due to NYU's Fall Break.

Midterm exam Oct 21

Lectures 8,9: The linear wave equation and related topics. Examples of problems leading to the wave equation, or closely related equations (including vibrating strings and membranes; acoustics; and Maxwell's equations). Solution formulas for the initial-value problem in R^n and in bounded domains. Domain of dependence (including multidimensional version via energy argument). Some basic numerical schemes (eg finite differences). Some non-constant-coefficient and nonlinear problems that can be done using similar techniques.

Lectures 10-11: Conservation laws and related topics. Examples of problems leading to scalar conservation laws and systems of conservation laws (eg traffic flow, shallow water flow). Burgers' equation (characteristics, shock formation, the Rankine-Hugoniot condition, admissibility condition for shocks, explicit solution via Hopf-Cole). Basic numerical schemes for scalar conservation laws. The method of characteristics, for conservation laws and for more general nonlinear first-order equations.

Lecture 12-13: Hamilton-Jacobi eqns. Examples of problems leading to Hamilton-Jacobi equations (optimal control, interface motion laws). Link to scalar conservation laws in 1D setting. Hopf-Lax solution formula. Solution by the method of characteristics. Basic numerical schemes. Brief discussion of viscosity solutions.

12/16: Final exam. Our final exam will be Tues 12/16 in the normal class slot and location (Tues 5:10-7pm, WWH 102).