PDE I
MATH-GA 2490, Fall 2013
Tuesdays 5:10-7:00pm, WWH 102

Final Exam information added 10/9/2013

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Prerequisites: A good knowledge of undergraduate-level linear algebra and ODE; also some exposure to complex variables (can be taken concurrently). This is an introductory course, but it will move quickly and require considerable mathematical maturity. The course is aimed mainly at entering PhD students, but is suitable also for well-prepared MS students.

Course description: A basic introduction to PDEs, designed for a broad range of students whose goals may range from theory to applications. This course emphasizes examples, representation formulas, and properties that can be understood using relatively elementary tools. We will take a broad viewpoint, including how the equations we consider emerge from applications, and how they can be solved numerically. Topics will include: the heat equation; the wave equation; Laplace's equation; conservation laws; and Hamilton-Jacobi equations. Methods introduced through these topics will include: fundamental solutions and Green's functions; energy principles; maximum principles; separation of variables; Duhamel's principle; the method of characteristics; numerical schemes involving finite differences or Galerkin approximation; and many more.

Homeworks, exams, grades: There will be weekly homework sets, an *in-class midterm exam on 10/22*, and a final exam (*Tues 12/17, in the normal class time and place: 5:10-7pm, WWH 102*). The semester grade will be based on the HW (1/3), the midterm (1/3), and the final exam (1/3).

Collaboration on homework is encouraged (homeworks are not exams) but registered students must write up and turn in their solutions individually. If you work with another student, please name him or her on your solution sheet. HW may be turned in late only by securing permission before it is due. Doing the HW is important, not only because it counts as part of the grade, but also because if you don't do the HW, you probably won't do well on the exams.

The midterm and final exams will be closed-book, but you may bring one sheet of notes (8.5×11, both sides, any font) to the midterm, and two sheets of notes to the final. Requests to take a makeup exam must be made in advance, and will *not* be granted for matters of personal convenience.

Recommended books: *(This list is long. See below, under "Which books to get?" for specific advice which books to download or buy.)* We will not follow any single book linearly. But to achieve appropriate mastery, you'll need to know more than just what we cover in class, and to see examples beyond what are assigned for homework. Here are some lists of books more basic than, similar to, and more advanced than this class. The Amazon prices listed below are for new books (in many cases used copies or prior editions cost less).

More basic than this class:

- W. Strauss, *Partial Differential Equations: An Introduction*, John Wiley and Sons, 1992. The best undergraduate-level text I know. Many of the topics we'll discuss are present in Strauss, at least in some measure, with an exposition that may be more accessible, especially if your PDE background is weak. I strongly recommend reading the relevant sections of this book alongside the more sophisticated texts below. (Amazon's price: about $70.)

- H. Weinberger, *A First Course in Partial Differential Equations, with Complex Variables and Transform Methods*, Dover, 1965. The first half is a lot like Strauss: a discussion of PDE making heavy use of separation of variables, but also emphasizing that there's much more to the theory than that. The second half is a good, application-oriented introduction to complex variables. Feels a little dated by now, but this material hasn't changed since 1965 and you can't beat the price. (Amazon's price: about $15.)

About the same level as this class:

- J. Ockendon, S. Howison, A. Lacey, and A. Movchan, *Applied Partial Differential Equations*, Oxford University Press, Revised Edition 2003. Very concrete and practical. Excellent discussions of the physical or probabilistic motivations for various PDE's. Plenty of problems. But: the order in which topics are discussed is very different from my plan. And the book tries to cover so many applications that it's like drinking from a firehose. (Amazon's price: about $62.)

- F. John, *Partial Differential Equations*, 4th edition, Springer-Verlag 1982. Spends less time than Ockendon et al discussing where the PDE's come from, and offers relatively few exercises. But the exposition is a model of clarity. *Downloadable for free from within the nyu.edu domain* from the website
  http://link.springer.com/book/10.1007/978-1-4684-0059-5/page/1
  (The same page will also offer you the opportunity to order an inexpensive soft-cover edition.)

- J. Kevorkian, *Partial Differential Equations: Analytical Solution Techniques*, 2nd edition, Springer-Verlag, 1999. Another excellent text at about the same level. Many good problems. *Downloadable for free from within the nyu.edu domain* from the site
  http://link.springer.com/book/10.1007/978-1-4757-3266-5/page/1

  You can also download John and Kevorkian from outside NYU, provided you have an NYU-Home account, by using the NYU proxy server; see
  http://cims.nyu.edu/webapps/content/systems/userservices/netaccess/proxy
  for information on that.

- R. Guenther and J. Lee, *Partial Differential Equations of Mathematical Physics and Integral Equations*, Dover, 1996. Yet another excellent text at this level. I like its discussions of where the PDE's come from. The treatment of boundary integral methods for Laplace's equation is among the best I know. Great value for money. (Amazon's price: about $15.)

- Q. Han, *A Basic Course in Partial Differential Equations*, American Mathematical Society, 2011. Starts (like many books) with first-order eqns. Most of the book focuses on the linear heat, wave, and Laplace equations, viewed from many viewpoints (same approach as we'll take in this class). Good problems. Relatively little about nonlinear PDE (except first-order eqns and the method of characteristics). Overall: a lot of overlap with this class, covered at about the same level. (Amazon's price: about $60.)

- P. Garabedian, *Partial Differential Equations*, 2nd revised edn, AMS Chelsea Publishing, 1998. Excellent treatments of many basic topics (eg the heat eqn, the wave eqn, Laplace's eqn, first-order eqns) as well as some less basic topics (eg integral equations, application of PDE to fluid dynamics, finite-difference-based numerical methods). Like John, this text has stood the test of time. (Amazon's price: about $41.)

Books that start at our level, but then go more advanced:

- L.C. Evans, *Partial Differential Equations*, American Mathematical Society, 2nd edition, 2010. Chapters 2 (Four important linear pde), 3 (Nonlinear first-order pde), and 4 (Other ways to represent solutions) are largely at the level of this class. Then the book continues with more advanced material (at the level of PDE II). Evans is especially good for mathematical aspects of scalar conservation laws, for the link between optimal control and Hamilton-Jacobi equations, and for material on viscosity solutions. (Amazon's price: about $65.)

- G. Folland, *Introduction to Partial Differential Equations*, 2nd edition, Princeton University Press, 1995. Chapters 2 (The Laplace Operator), 4 (The heat operator), and 5 (The wave operator) are largely at the level of this class. The rest of the book is more advanced (at the level of PDE II). Folland is an excellent complement to Evans, since it covers single and double layer potentials (which Evans omits) and it relies largely on Fourier-transform-based methods (which Evans de-emphasizes). (Amazon's price: about $75.)

- M. Renardy and R. Rogers, *An Introduction to Partial Differential Equations*, 2nd edition, Springer-Verlag, 2004. This book spends even less time on our material than Evans or Folland, and covers it less systematically. The book is mainly more advanced, at the level of PDE II. One distinguishing feature is its treatment of evolution equations, which has separate chapters on energy-based and semigroup-based techniques. (Amazon has this for around $70.)

All these books will be on reserve in the CIMS library. (But: Ockenden et al was missing and had to be ordered; it may take a while to arrive.) For thoughtful reviews by J. David

Logan that discuss and compare some of these books, see SIAM Review 42 (2000) 515-522 and SIAM Review 51 (1999) 393-395.

Which books to get? Of course nobody should buy all the books listed above. My advice is to get Kevorkian and John (both available as free downloads), and to buy the book by Guenther & Lee (which complements Kevorkian and John nicely, since it has somewhat emphasis and scope).

If you plan on going on in PDE (for example, taking PDE II) you'll probably want to buy Evans eventually. In that case it might be worth buying it now, since the first 4 chapters correlate pretty strongly with this class.

Tentative semester plan:

9/3, 9/10, 9/17: The heat equation and related topics. Examples of problems leading to the heat equation or closely related equations (probability as well as physics). Solution formulas for the Cauchy problem, half-space problems, and bounded domains. Backward eqn is ill-posed. Uniqueness via energy argument and via maximum principle. Some basic numerical schemes (finite differences, Galerkin approximation, explicit vs implicit time-stepping). Some nonconstant-coefficient and nonlinear problems that can be done using similar techniques.

9/24, 10/1, 10/8: Laplace's equation and related topics. Examples of problems leading to Laplace's equation, or closely related equations (including probability and fluid dynamics). Solution formulas using complex variables, the fundamental solution, separation of variables, and the Green's function. Uniqueness via energy argument and via maximum principle. Variational principles. Some basic numerical schemes (finite differences, finite elements, Galerkin approximation). Some non-constant-coefficient and nonlinear problems that can be done using similar techniques.

No class Oct 15, due to NYU's Fall Break.

Midterm exam Oct 22

10/29, 11/5: The linear wave equation and related topics. Examples of problems leading to the wave equation, or closely related equations (including vibrating strings and membranes; acoustics; and Maxwell's equations). Solution formulas for the initial-value problem in R^n and in bounded domains. Domain of dependence (including multidimensional version via energy argument). Some basic numerical schemes (eg finite differences). Some non-constant-coefficient and nonlinear problems that can be done using similar techniques.

11/12, 11/19: Conservation laws and related topics. Examples of problems leading to scalar conservation laws and systems of conservation laws (eg traffic flow, shallow water flow). Burgers' equation (characteristics, shock formation, the Rankine-Hugoniot condition, admissibility condition for shocks, explicit solution via Hopf-Cole). Some discussion of analogous issues for shallow water flow. Basic numerical schemes for scalar conservation laws.

11/26: Hamilton-Jacobi eqns. Examples of problems leading to Hamilton-Jacobi equations (optimal control, interface motion laws). Link to scalar conservation laws in 1D setting. Hopf-Lax solution formula. Brief discussion of viscosity solutions. Solution by the method of characteristics. Basic numerical schemes.

12/3, 12/10: Characteristic curves and non-characteristic surfaces. Solution of first-order equations by the method of characteristics. Non-characteristics surfaces and the Cauchy-Kowalewsky theorem. Connections with material discussed earlier in the semester.

12/17: Final exam. Our final exam will be Tues 12/17 in the normal class slot and location (Tues 5:10-7pm, WWH 102).