Structural Optimization and 3D Printing

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SIAM CS&E Meeting, March 2015

Outline

Mandate: identify areas with interesting open questions. Hence 3D printing and structural optimization.

My involvement began with an unexpected call from Denis Zorin ...

Plan for the talk:

- 3D printing
- Recent work by D Zorin, J Panetta, Q N Zhou, & others
- A potential application: bone scaffolds
- Some potential research directions

Related to my DMREF project: *Adaptive fine-scale structure design: from theory to fabrication*, DMS-1436591 (D. Zorin, J. Ricci, and Y. Zhang co-PI's)

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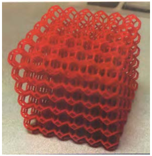
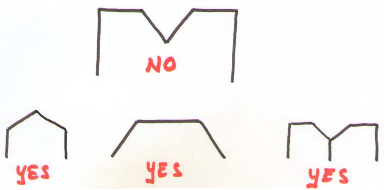
3D printing

There are multiple technologies; focus here on stereolithography: structure is built up one layer at a time, pixel by pixel.



Cost of manufacture is independent of complexity, but

- finite resolution (determined by pixel size)
- each layer must be supported by the one below it



Which Hooke's laws are achievable?

In 1995, Cherkaev & Milton asked: Can any Hooke's law be achieved by a suitable microstructure? Their answer: yes. Related questions involving 3D printing:

- Can any Hooke's law be achieved manufacturably?
- Find a convenient correspondence between Hooke's laws and manufacturable microstructures.

The literature is thin, but includes:

(a) 3D printing of truss-like structures with neg Poisson ratio (Buckmann et al, Adv Mater 24, 2012, 2710)

(b) 3D printing of periodic truss-like structures with high rigidity (X Zheng et al, Science 344, 2014, 1373)

(c) work on truss-like structures with maximal rigidity (Gurtner & Durand, Proc Roy Soc A 470, 2014, 20130611, cf also Bourdin & Kohn, JMPS 56, 2008, 1043)

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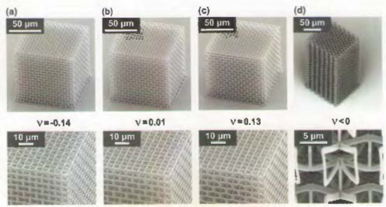

3D printing of high-strength cellular ceramic composites
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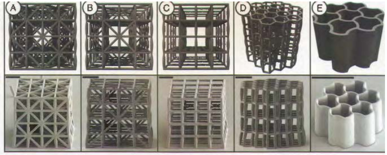

- 3D printing
- Recent work by D Zorin, J Panetta, Q N Zhou, L. Malmo, N. Pietroni, and P. Cignoni
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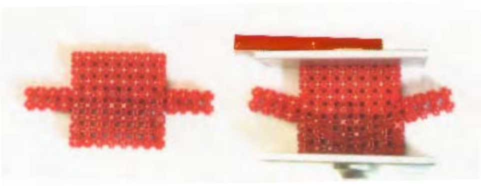
Designing a structure with a specific response

Goal: Use 3D printing to make a structure with a specified response to a given load.



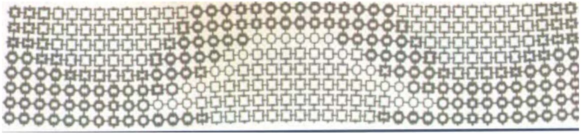
More mathematically: given a region Ω, a traction $f : \partial\Omega \to R^3$, and a desired displacement $u : \partial\Omega \to R^3$, print a light-weight, structure with overall shape Ω for which the boundary load f produces the boundary displacement u.

Designing a structure with a specified response

Work by D. Zorin with PhD students J. Panetta, and Q.N. Zhou:

STEP 1 Find a continuously-varying Hooke's law $C_{ijkl}(x)$, defined on Ω, whose elastic response has the desired character.

STEP 2 Realize $C(x)$ by a locally periodic composite whose cell structure varies with x.



Implementation: choose topology of period cell and keep it fixed, so neighboring cells connect properly despite their varying structure.

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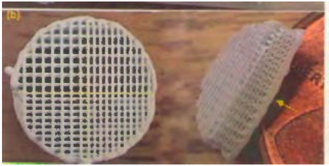
Bone scaffolds

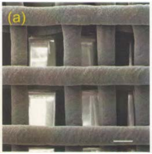
Goals range from *preparing jaw for a tooth implant* to *repair of load-bearing structures*.

Typical material is hydroxyapetite (a ceramic that's friendly to bone).

Typical technology is "direct-write printing" (moving nozzle leaves behind a cylinder of ceramic).

A simple structure for dental applications: a "log-cabin" type pattern



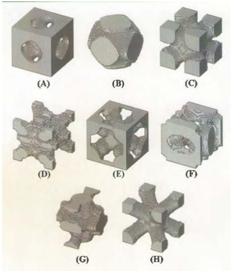
figs from: J Simon et al, J Biomed Mater Res A, DOI:10.1002/jbm.a.31329

Bone scaffolds

What matters?

- Structure must be porous, since bone precursors need to enter. Size of pores influences type of bone that grows.
- Often, boundary must be solid where it meets soft tissue, to keep the soft tissue from growing in.
- For dental applications, rigidity is unimportant; but for load-bearing structures, rigidity is crucial.

Scott Hollister's group has used numerical structural optimization to explore tradeoff btwn rigidity and porosity, using eff bulk modulus and eff diffusivity as proxies (eg H Kang et al, Struct Multidisc Optim 42, 2010, 633)



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Some research directions – linear elasticity

- In context of two-step approach discussed earlier, we need a better map from (macroscopically-varying) Hooke's laws to (locally periodic) manufacturable structures.
- There are other approaches to structural optimization, for example using the level set method (eg G Allaire, F Jouve, A Toader, J Comp Phys 194, 2004, 363) or using penalization to eliminate composites (eg M Bendsoe's book, Springer-Verlag, 2005). Can they be made to cooperate with manufacturability?

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Some research directions – beyond linear elasticity

- Failure by buckling. Structures that seem linearly stiff may still be prone to buckling. It is natural to ask that a structure be manufacturable and also not buckle under the anticipated loads. Or: one could ask that it buckle in a particular way. (eg SH Kang et al, PRL 112, 2014, 098701).
- Fracture toughness. In simple examples, heterogeneity seems to improve fracture toughness (eg Hossain et al, JMPS 71, 2014, 15). The essential reason: when a growing crack enters a more compliant phase, the stress singularity at the crack tip is reduced. Can we formulate and solve optimal design problems with fracture toughness as a design objective?
- Plastic limit load. My work in the 80's with Strang includes some simple examples of weight minimization with a constraint on the limit load. Can they be extended, and/or combined with manufacturability constraints?

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Stepping back

- With 3D printing, complexity is not a problem. But there are still constraints of manufacturability.
- For bone scaffolds, 3D printing has promise. However it is still not easy to say exactly how mathematics can help.
- For the analysis and design of high-porosity structures, there are still many open questions.
- Discussions that used to seem academic may now be quite practical using 3D printing.