Some possible presentation topics

Students registered for Mechanics in Spring 2022 must give a 25-minute presentation on a topic related to this course's material. This document provides 6 suggestions related to elasticity, and one related to Hamiltonian mechanics.

You are not required to choose from these suggestions! The books listed in our syllabus include many examples and/or topics that would be suitable for a presentation. Something related to your prior work or research interests will be fine, provided that it bears a reasonable relation to this class's scope and that you can present it in a way that's accessible to the class.

Please let me know your tentative choice of topic by Tuesday, April 5. If two or more students select a topic from this document, we'll have to discuss whether that subject is suitable for two distinct presentations. We will use our regular class slots on Tues May 10 and Tues May 17 for the presentations.

It can be difficult to craft a 25-minute presentation; its goals should be narrow enough to fit the limited timeslot, yet ambitious enough to be interesting. I am of course available for consultation.

Hair: Hair can be modelled as an inextensible one-dimensional elastic rod. This is developed in Chapter 4 of *Elasticity and Geometry: from hair curls to the nonlinear response of shells*, by B. Audoly and Y. Pomeau (available online through Bobcat).

Adhesion: A basic "adhesion" problem involves peeling a sticky tape from a surface by lifting one end. The tape can be modeled as an elastica, but now there is a free boundary (where it meets the surface). Recently some papers have explored what happens if the strength of the adhesive is nonuniform. *Adhesion of heterogeneous thin films I: Elastic heterogeneity* by S.M. Xia, L. Ponson, G. Ravichandran, and K. Bhattacharya (J Mech Phys Solids 61, 2013, 838–851) provides a good summary of the topic's history, and studies the consequences of heterogeneity in a simple (but already interesting) setting where a 1D model is possible.

Thin elastic ribbons: The mechanics of elastic ribbons (including, for example, a Mobius band) can be reduced to the study of a one-dimensional variational problem by writing the bending energy of the ribbon in terms of the curvature and torsion of its midline. The paper *A corrected Sadowsky functional for inextensible elastic ribbons* by L Freddi, P. Hornung, M.G. Mora, and R. Parroni, J Elasticity 123 (2016) 125-136 identifies the variational problem. (It also gives references to earlier work; you may find it more convenient to present one of the earlier papers.)

Uniqueness in nonlinear elasticity: In linear elasticity, solutions are unique when the displacement is specified on part or all of the boundary. In nonlinear elasticity, we know that structures can buckle so solutions are in general not unique. But it is natural to expect that the linear argument should extend to the nonlinear setting if the nonlinear strain is sufficiently small. Fritz John proved a theorem of this type in

1972, in his paper *Uniqueness of non-linear elastic equilibrium for prescribed boundary displacements and sufficiently small strains*, CPAM 25 (1972) 617–634. Its arguments are elegant and elementary, except for the use of a fundamental result about deformations with uniformly small strain (which could, for the purposes of a presentation, be simply taken as fact).

The nonlinear Korn inequality: In linear elasticity, Korn's inequality says that if the linear elastic energy of a deformation is small, then the deformation is close (in H^1) to a rigid motion. A nonlinear version of Korn's inequality lies at the heart of work by Friesecke, James, and Müller concerning the relation between 3D elasticity and Kirchhoff plate theory. Their proof of the nonlinear Korn inequality is elegant and self-contained, making it suitable for a presentation. The Friesecke-James-Müller paper is *A theorem on geometric rigidity and the derivation of nonlinear plate theory from three-dimensional elasticity* (CPAM 55, 2002, 1461–1506) and the nonlinear Korn inequality is presented in Section 3 (pp 1468–1474).

Fracture mechanics: In fracture mechanics, a widely-used model says that a crack will propagate if its "energy release rate" is larger than a critical value. The energy release rate is the rate at which the elastic energy decreases as the crack gets longer. It turns out to have a simple formula involving a path-independent integral. The following paper gives a careful account of this fact: M. Gurtin, *On the energy release rate in quasi-static elastic crack propagation*, J Elasticity 9 (1979) 187-195

Time crystal dynamics: The variational problem $\int_0^1 (u_x^2 - 1)^2 + u^2\,dx$ prefers $u_x = \pm 1$ and $u = 0$. Its minimum value is 0, as one sees by considering piecewise linear u with $u_x = \pm 1$ in the limit as the length scale of its slope oscillation tends to 0. In Hamiltonian mechanics, the Lagrangian leads (as we'll see very soon) to one-dimensional variational problems involving functions of *time*. The recent paper *Regularizations of time-crystal dynamics* by A Shapere and F Wilczek, PNAS 116(38), 2019, 18772–18776 considers a mechanical system whose Lagrangian variational principle is reminiscent of the example just mentioned.