MECHANICS
MATH-GA.2710-001, Spring 2019
Wednesdays 1:25-3:15pm, WWH 517

Instructor: Robert Kohn (kohn@cims.nyu.edu, 8-3217, WWH 502). Office hours: Tues 2-3 and Wed 3:30-4:30 (or by appt).

Description: This course provides brief introductions to elasticity, classical mechanics, and statistical mechanics – topics at the interface where ODE, PDE, and probability meet physics and materials science. For students preparing to do research on physical (or biophysical) applications, the class provides an introduction to crucial concepts and tools; for students of analysis and PDE the class provides valuable context by exploring some important applications. No prior exposure to mechanics or physics is assumed. The scope of the material to be discussed is vast, so we must make some choices. We'll spend just over half the semester on elasticity, followed by shorter segments on classical mechanics & statistical mechanics:

- Our discussion of *elasticity* (about 7 weeks) will start with one-dimensional models (strings and rods) and buckling (as an example of bifurcation); then we'll continue with finite elasticity and linear elasticity in the context of 3D solids.
- Our discussion of *classical mechanics* (about 3 weeks) will start with Hamilton's equations and action minimization as alternative formulations; then we'll explore connections with the calculus of variations and PDE (including geodesics, Hamilton-Jacobi equations & optimal control, and some nonlinear wave equations).
- Our discussion of *statistical mechanics* (about 3 weeks) will start with basic concepts such as the microcanonical and canonical ensembles; then we'll discuss Metropolis sampling (as an introduction to Markov Chain Monte Carlo) and the Ising model (as an example of phase transition).

One viewpoint on this class is that it complements the Fall semester Fluid Dynamics course, by providing an introduction to some other areas of mechanics. (Please note, however, that Fluid Dynamics is *not* a prerequisite for this class.)

Calendar: The first lecture is Wed 1/30. There will be no class Wed 3/20 (spring break) or Wed 4/24 (I'll be at a meeting). The last lecture is Wed 5/8. The finals-week slot (Wed 5/15) will be used for student presentations.

Prerequisites: The minimal prerequisites are ODE and PDE at the level of an undergraduate class (or preferably, in the case of PDE, at the level of Introduction to Partial Differential Equations, GA2490-001). However this course also demands substantial mathematical maturity: while each segment starts at the beginning, we will move very quickly. The course has been designed with first and second year PhD students in mind; Masters students should consult with the instructor before registering.

Requirements: The course requirements are two-fold: (1) problem sets, and (2) a 25-minute presentation. The problem sets will reinforce recently-covered material by exploring applications and examples; you are welcome (even encouraged) to discuss the problems with others, but each student must write up his/her solutions individually. The 25-minute presentations can be on any topic within the course's scope; some suggestions related to our elasticity segment will be distributed around spring break. Students should choose their presentation topics by the beginning of April. We will use the finals-week class slot (May 15) for presentations; if (as seems likely) more time is needed, additional presentation sessions will be scheduled during the last week of classes or finals week. There will be no final exam.

Websites: I will put scanned notes and homework assignments on a public website https://math.nyu.edu/faculty/kohn/mechanics.html . (This site also has a link to my notes and HW from Spring 2018, when I last taught this class; the Spring 2019 segment on elasticity will be similar to 2018, but the classical mechanics and statistical mechanics segments will be somewhat different.) The course will also have an NYU Classes site, for material that's intended only for registered students, and for communication by email.

Books: We will not follow any text linearly. But (I hope) you'll want to know more about each topic than we can possibly cover in a few lectures. Therefore the following books might be useful (all will be on reserve in the Courant library).

Elasticity

- P. Howell, G. Kozyreff, J. Ockendon, *Applied Solid Mechanics*, Cambridge Univ Press 2009 (available electronically through Bobcat).
- G. Duvaut and J.-L. Lions, *Inequalities in mechanics and physics*, Springer, 1976 (available electronically through Bobcat).
- S. Antman, *Nonlinear problems of elasticity*, Springer-Verlag, 2005 (available electronically through Bobcat).
- P.G. Ciarlet, *Mathematical elasticity I: Three-dimensional elasticity*, North-Holland, 1988 (available electronically through Bobcat).
- J.E. Marsden and T.J.R. Hughes, *Mathematical foundations of elasticity*, Prentice-Hall, 1983 (now available inexpensively from Dover).

Howell-Kozyreff-Ockendon provides a wide-ranging introduction at a relatively basic level; if you're new to this topic, it could be worth reading cover-to-cover. The other books are much more advanced (not recommended for cover-to-cover reading). They are in fact quite different from one another. Duvaut & Lions is very "French" (emphasizing weak solutions, for example), with a focus on physically nonlinear but geometrically linear models. Antman by contrast emphasizes geometrically nonlinear models. Ciarlet provides the basics of nonlinear elasticity in a clean, well-organized, but rather dry form. The first 25 pages of Marsden & Hughes gives a concise introduction to nonlinear elasticity.

Classical mechanics

- O. Buhler, *A brief introduction to classical, statistical, and quantum mechanics*, Courant Lectures Notes in Mathematics vol 13, 2006.
- V. I. Arnol'd, *Mathematical methods of classical mechanics*, Springer (available electronically through Bobcat).
- L.D. Landau and E.M. Lifshitz, *Mechanics.* (The 3rd edition is available electronically through Bobcat).
- J.V. José and E.J. Saletan, *Classical dynamics: A contemporary approach*, Cambridge University Press, 1998 (available electronically through Bobcat).

Another source – not in the CIMS library (so not on reserve and therefore not listed above) but available online through the Springer book package – is E. DiBenedetto, *Classical Mechanics: Theory and Mathematical Modeling*, Birkhauser, 2011.

Buhler's book has the advantage of conciseness, but the disadvantage of having few examples or exercises – overall: highly recommended for its big-picture perspective, but not really sufficient. Landau & Lifshitz (a classic) and José & Saletan (more modern) have many more examples and exercises; I prefer the latter due to its more modern and geometric perspective. DiBenedetto's book seems to be at about the same level as José & Saletan. Arnol'd's book is deeper and more advanced; while it does cover material similar to ours in the first few chapters, most of the book discusses a geometric side of the subject that we won't have time to get to.

Statistical mechanics

- O. Buhler, *A brief introduction to classical, statistical, and quantum mechanics*, Courant Lectures Notes in Mathematics vol 13, 2006.
- A. Chorin and O. Hald, *Stochastic tools in mathematics and science*, 3rd edition, Springer, 2013 (available electronically through Bobcat).
- D. Chandler, *Introduction to modern statistical mechanics*, Oxford University Press, 1987.
- M. Tuckerman, *Statistical mechanics: theory and molecular simulation*, Oxford Univ Press 2010 (available electronically through Bobcat) .

The comment made earlier about Buhler's book applies here too. Chapters 7 & 8 of Chorin & Hald are similarly concise, with a choice of topics somewhat different from Buhler's (and closer to my own taste). For more detailed introductions, the books by Chandler and Tuckerman are both highly recommended, though they are somewhat different from one another. Chandler starts with a 50-page introduction to thermodynamics (which can however be skipped); chapters 3-6 couple well to this class, though of course they do much more than we'll have time for. Tuckerman starts with a 50-page introduction to classical mechanics (not unlike our segment on Classical Mechanics); chapters 3, 4 and 7 couple to this class, though again they go far beyond what we'll have time for.