Special Topics in Applied Math
Mathematical Models of Crystal Growth
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G63.2840.004, Spring 2007
Tuesdays 5:10-7:00pm, WWH 813
Revised 1/16/07

Description: Crystal growth is an intrinsically multiscale problem: the basic mechanisms of growth are atomic in nature and therefore discrete, but their macroscopic consequences are best expressed by PDE's. This course will introduce students to some widely-used models, and to interesting open questions about the relations between them. Topics will include: the Burton-Cabrera-Frank model; 1D and radial step motion laws, and their continuum limits; steepest-descent (thermodynamic) PDE models; and stochastic approaches. A central theme is the task of coarse-graining discrete models, i.e. connecting them with PDE's that describe their macroscopic behavior.

Prerequisites: General-exam-level knowledge of ODE's and PDE's. No knowledge of physics will be assumed.

Requirements: There will be no problem sets or exams (this is a "seminar course.") Students may volunteer to give presentations, but are not required to do so.

Website: I'll activate a Blackboard site (let me know if you'd like access though you're not registered for the class), for posting scanned lecture notes and similar materials in a not-too-public way.

Library reserve: The following books will be on reserve in the Courant library:

1. A. Pimpinelli and J. Villain, *Physics of Crystal Growth*, Cambridge University Press 1998 is the best introductory text I know. The recent
2. T. Michely and J. Krug, *Islands, Mounds, and Atoms: Patterns and Processes in Crystal Growth Far from Equilibrium*, Springer 2004

Pimpinelli and Villain is meant as an introduction for physicists. Michely and Krug is more of a research monograph. Students with no background in physics will find these hard to read at the beginning of the semester (hopefully less so by the end of the semester).

Other sources: The physics literature is huge. Here are two downloadable review articles:

(a) H-C Jeong and E.D. Williams, *Steps on surfaces: experiment and theory*, Surface Science Reports 34 (1999) 171-294 (available through ScienceDirect)

(b) J.W. Evans, P.A. Thiel, and M.C. Bartelt, *Morphological evolution during epitaxial thin film growth: formation of 2D islands and 3D mounds*, Surface Science Reports 61 (2006) 1-128 (available through ScienceDirect).

The mathematics literature is much more sparse, and I don't know any review articles. But don't be scared by that. All lectures will be supported by either lecture notes or suggested readings.

Semester plan

Lecture 1, 1/16 Brief overview: different types of models (stochastic vs pde) and different types of surface evolution (island dynamics vs step dynamics; deposition and evaporation vs surface-diffusion-driven coarsening). Introduction to step dynamics. *(Kohn)*

Lecture 2, 1/23 Step dynamics continued. Formal coarse-graining of 1D models for deposition and for relaxation. *(Kohn)*

Lecture 3, 1/30 Step dynamics continued. Formal coarse-graining in the radial setting and fully-two-dimensional settings. Discussion of peaks and valleys. *(Kohn)*

No class 2/6 I'm out of town this week.

Lecture 4, 2/13 Introduction to island growth: KMC simulation, continuum methods, and a hybrid numerical scheme. *(Yi Sun)*

Lecture 5, 2/20 Crystalline anisotropy: the static and dynamic Wulff shapes. *(Kohn)*

Lecture 6, 2/27 Introduction to the thermodynamic ("steepest-descent") viewpoint on relaxation. *(Kohn)*

Lecture 7, 3/6 Analysis of steepest-descent pde's. *(Kohn)*

No class 3/13 NYU's spring break

No class 3/20 I'm out of town this week.

Lecture 8, 3/27 Theoretical results on some steepest-descent pde's. *(Jungho Lee)*

Lecture 9, 4/3 Numerical analysis of some steepest-descent pde's. *(Henrique Versieux)*

Lecture 10, 4/10 Topics in step dynamics: Krug's "wedding cakes" and/or the Israeli-Kandel scaling laws. *(Hala Al Hajj Shehadeh)*

Lecture 11, 4/17 To be determined.

No class 4/24 There's a meeting on epitaxial growth at the University of Maryland this week. Registration is free but required, due to space limitations. See http://www.cscamm.umd.edu/programs/nid07 for information.

Lecture 12, 5/1 To be determined (note this is officially "reading day").