Derivative Securities
MATH-GA 2791.001, Fall 2012
Wednesdays 7:10-9:00pm
16-18 Waverly, room G-08
Revised 10/17 by changing homework plan

Location: 16-18 Waverly is the new Genomics & Systems Biology building, near the corner of Waverly and Mercer. Be sure to bring your NYU ID.

Instructor: Robert V. Kohn. Office: 502 Warren Weaver Hall. Phone: 998-3217. Email: kohn@cims.nyu.edu. Web: math.nyu.edu/faculty/kohn. Office hours: Mon 5-6 and Wed 11-12, starting 9/10.

Grader: Xingxin Zhong, email xingxin.zhong@cims.nyu.edu, office WWH 1111. Office hour: Wed 5-6 in WWH 1111.

Course description: An introduction to arbitrage-based pricing of derivative securities. Topics include: arbitrage; risk-neutral valuation; the log-normal hypothesis; binomial trees; the Black-Scholes formula and applications; the Black-Scholes partial differential equation; American options; one-factor interest rate models; swaps, caps, floors, swaptions, and other interest-based derivatives; credit risk and credit derivatives.

Special dates: Lecture 1 (Sept 5) will be given by Steve Allen. There will be no lecture on Wednesday Sept 26 (Yom Kippur). There *will* be a lecture on Wed Nov 21 (the evening before Thanksgiving); it will combine a semester review (intended to help you prepare for the exam) with some enrichment material (not covered by the exam). The last class will be Wed Dec 5. Our Final Exam is Wed Dec 19, at the usual class time and place.

Lecture notes: Lecture notes, homework assignments, etc. will be posted on my web page in pdf format. I'll be refreshing the notes as we go along, but at the top of the course website you'll find a link to the page built the last time I taught this course, in Fall 2007. I will also maintain a Blackboard page, as a communication tool and for material whose distribution is limited to students in the class. If you are not registered for the class but wish to be added to the BlackBoard site, ask me.

Prerequisites: Multivariable calculus, linear algebra, and discrete probability. If your mathematical background needs brushing up, the book *A Primer for the Mathematics of Financial Engineering*, by Dan Stefanica, might be useful. Concerning probability: students should be familiar with concepts such as expected value, variance, independence, conditional probability, the distribution of a random variable, the Gaussian distribution, the law of large numbers, and the central limit theorem. These topics are addressed early in most undergraduate texts on probability.

Homeworks, final exams, grades: There will be 7 homework sets, one every couple of weeks. Collaboration on homework is encouraged (homeworks are not exams) but registered

students must write up and turn in their solutions individually. If you worked with another student, please name him or her on your solution sheet. HW may be turned in late only by securing permission before it is due. The final exam will be closed-book, but you may bring two sheets of notes (8.5×11, both sides, any font). Requests to take a makeup exam must be made in advance, and will *not* be granted for matters of personal convenience. The semester grade will be based 2/3 on the final, 1/3 on the homework. (But: if you don't do the homework regularly you probably won't do well on the final.)

Textbook: The main text for this class is:

- J.C. Hull, *Options, futures and other derivatives*, 8th edition, Prentice Hall, 2012.

The 8th edition includes many changes related to the financial crisis of 2008 and other recent market developments. While it will be possible to make do with an earlier edition, I recommend getting the current one. Some notes: (i) Hull provides a lot of practical information that we won't be discussing (e.g. how various markets work). (ii) Our discussions will often be considerably more mathematical than Hull's (thus the Lecture Notes are more or less a second text). (iii) We won't proceed linearly through Hull, and we won't cover some of the more advanced topics found there.

Other recommended books: Here are some additional books you may wish to consult. All 6 are on reserve in the CIMS library.

Particularly recommended (books you may wish to purchase):

- M. Joshi, *The concepts and practice of mathematical finance*, Cambridge Univ Press, 2nd edition, 2008. The first 13 chapters are a lot like Lectures 1-10 of this class. Then Joshi moves to more advanced material on options, while we turn instead to credit.

- M. Baxter and A. Rennie, *Financial calculus: an intoduction to derivative pricing*, Cambridge University Press, 1996. The first 50 pages provide a fine introduction to binomial trees. The rest of the book works uses continuous-time models, and correlates well with the initial part of the class Continuous Time Finance.

Also recommended:

- K. Back, *A course in derivative securities: introduction to theory and computation*, Springer, 2005 (available in paperback starting 2010). Aims at roughly the same level as this class, but takes a very different approach to many topics (e.g. Back does not start with binomial trees).

- S. Neftci, *An introduction to the mathematics of financial derivatives*, Academic Press, 2nd edition (2000). For students who have not taken Stochastic Calculus, this book offers an accessible introduction to stochastic differential equations and the Ito calculus, going beyond what we'll do in this class.

- P. Wilmott, S. Howison, and J. Dewynne, *The mathematics of financial derivatives - a student introduction*, Cambridge University Press (1995). This book underplays the role of probability and overplays the role of PDE. Still, for students with some background in PDE it offers an alternative perspective.

- R. Jarrow and S. Turnbull, *Derivative Securities*, Southwestern Publishing, 2nd edition, 2000. Similar to Hull in its goals and spirit. Somewhat out of date by now, but still very useful for the more basic material. (For example, the topics we cover in Lecture 1 are distributed across many chapters of Hull, but are presented in a more concise and focused way in Jarrow & Turnbull.)

Semester plan:

Lecture 1, 9/5: Forwards and options; absence of arbitrage and its consequences. HW1 distributed.

Lecture 2, 9/12: Single-period trees.

Lecture 3, 9/19: Multiperiod trees (European and American options). HW1 due, HW2 distributed.

No class 9/26: Yom Kippur

Lecture 4, 10/3: Lognormal dynamics; derivation of the Black-Scholes formula via continuum limit of binomial trees. HW2 due.

Lecture 5, 10/10: Properties and uses of the Black-Scholes formula, including "the greeks," hedging, and implied volatility. HW3 distributed.

Lecture 6, 10/17: The continuous-time viewpoint: introduction to SDE and Ito's formula; the Black-Scholes pde.

Lecture 7, 10/24: Martingales and risk-neutral expectation; barrier and more exotic options. HW4 distributed.

Lecture 8, 10/31: Introduction to interest-based instruments (bonds, the yield curve, swaps, caps, floors, swaptions). HW3 due.

Lecture 9, 11/7: Pricing of options on interest-based instruments, using Black's formula and using trees. HW4 due. HW5 distributed.

Lecture 10, 11/14: Short-rate models; justification of Black's formula (change of numeraire).

Lecture 11, 11/21: Review of material to date, plus something fun (which won't be on the exam). Note: this is the evening before Thanksgiving.

Lecture 12, 11/28: Credit risk. HW5 due. HW6 distributed.

Lecture 13, 12/5: Credit derivatives.

No class 12/12: Legislative day – Monday classes meet this day. HW6 due.

Final Exam 12/19: You may bring two sheets of notes (8.5×11, any font) to the exam. Requests to take a makeup must be made in advance, and will *not* be granted for personal convenience.