Derivative Securities – Fall 2012 – Final Exam Guidance
Extended version – includes full semester

- Our exam is Wednesday, December 19, at the normal class place and time.
- You may bring two sheets of notes (8.5 × 11, both sides, any font). No books, calculators, or computers are permitted.
- Some of the questions will be similar to (pieces of) homework problems you have done. Others will address crucial concepts or calculations discussed in the notes and lectures. For guidance what type of questions to expect, see my Fall 2004 Derivative Securities final, which is at www.math.nyu.edu/faculty/kohn/derivative_securities_2004.html . (Note: there are underscores after "derivative" and "securities".)
- Material in the Section 10 notes (corresponding to the 11/21 lecture) will not be on the exam. Also, the justification of Black's formula for interest-based options (which is in the Section 9 notes, but was discussed in class on 11/21) will not be on the exam. The *use* of Black's formula for interest-based options will, however, be on the exam.
- Material in the Section 12 notes (corresponding to the 12/5 lecture) will not be on the exam.

Here are some examples of things that could be on the exam. Please note that *this list is not complete*: other topics of comparable importance could also be on the exam.

Section 1: Forwards, puts, calls, and absence of arbitrage

Concepts:

- Be able to explain how investors make use of forwards and options markets to take positions on assets they think will increase (or decrease) in value without needing to invest money.
- Be able to explain how a few investors seeking arbitrage profits can ensure that "no arbitrage" relationships hold for all investors.
- Be able to explain the difference between the expected value of an asset and its forward price.

Computations:

- Given the price of an asset (stock, foreign exchange, commodity), the risk free rate, and the borrowing rate for the asset, be able to compute the forward price (the delivery price that makes the present value of the forward contract 0). If the actual forward price differs from this computed price, determine what action is required to make a riskless profit and calculate the profit.

- Be able to derive the borrowing rate of a commodity from the spot and forward price.
- Be able to find a portfolio of options with a given payoff diagram.
- Be able to determine actions necessary to profit from options prices that are inconsistent (e.g. due to failure of put-call parity), and to calculate the resulting profit.

Proofs:

- Be able to prove the put-call parity formula.

Sections 2 and 3: binomial trees

Concepts:

- Be able to explain why early exercise is never optimal for an American call on a non-dividend-paying stock. Also, be able to show that for an American call on a dividend-paying stock, and for an American put on a non-dividend-paying stock, early exercise can be optimal in some circumstances.
- Be able to explain why the futures price is a martingale under the risk-neutral probability.

Computations:

- Be able to value a European call or put option on stock price or forward price, by working backwards through a binomial tree. (This includes finding the risk-neutral probabilities associated with the tree.) Be able to calculate an appropriate hedge (i.e. a position in stock or futures or forwards that eliminates your risk) at each node of the tree. Be able to replicate the option using stock, forwards or futures.
- Be able to value an American call or put using a binomial tree, and to find an appropriate hedge at each node.

Proofs:

- Be able to explain why, in a multiplicative stock price tree, the risk-neutral probability of the up branch satisfies $q = \frac{e^{r\delta t}-d}{u-d}$, and in a multiplicative forward price tree it satisfies $q = \frac{1-d}{u-d}$.

Sections 4 and 5: the Black-Scholes formula and its applications

Concepts:

- Be able to sketch the logic by which we found that for a lognormal non-dividend-paying stock, the risk-neutral distribution of stock prices is $s_t = s_0 e^X$ where X is Gaussian with mean $(r - \frac{1}{2}\sigma^2)t$ and variance $\sigma^2 t$. (You would not be asked to reproduce the argument in full detail.)
- Be able to provide intuition behind the signs of the Greeks for calls and puts.

- Be able to explain why the price of an option uniquely determines an implied volatility.
- Be able to explain how "fat tails" are related to the dependence of implied volatility on the strike of an option.

Computations:

- Given the drift and volatility of a lognormal stock price, give an expression (for example) for the probability that the price will lie in a certain interval at time T.
- Be able to explain how our formula for $E\left[e^{aX} \text{ restricted to } X > k\right]$ (with X Gaussian) leads to the Black-Scholes formulas for puts and calls. Or use this formula to price a different option (as done in a couple of HW problems).
- Be able to calculate implied volatility using Newton's method.

Proofs:

- Explain why the Delta of a call is $N(d_1)$.
- Use put-call parity to explain the relationship between the Delta, Vega, or Gamma of a call and those of a put.

Sections 6 and 7: Stochastic differential equations, martingales, and exotic options

Concepts:

- Be able to say what it means to be a martingale, in both a binomial tree and continuous-time setting.
- Be able to say why the Black-Scholes PDE is useful for valuing European options, but not for most exotic options.
- Be able to say why the price of a non-dividend-paying stock under the risk-neutral probability satisfies (in a constant-interest-rate setting) $ds = rs\,dt + \sigma s\,dw$. Also why the forward price satisfies $dF = \sigma F\,dw$.

Computations:

- Be able to apply Ito's lemma, to find an SDE satisfied by $z = f(s(t), t)$, where s solves a given SDE and f is a given function of two variables. Be able to apply this, e.g. as we did in several HW problems.
- Be able to value path-dependent options (such as an Asian option, or a barrier option) using a tree.

Proofs:

- Be able to show that if $V(s,t)$ solves the Black-Scholes PDE $V_t + rsV_s + \frac{1}{2}\sigma^2 s^2 V_{ss} - rV = 0$ for $t < T$ with $V(s,T) = \phi(s)$, and $ds = rs\,dt + \sigma s\,dw$, then $V(t, s(t)) = e^{-r(T-t)}E[\phi(s(T))]$. at

- Be able to show that if $V(s,t)$ solves the Black-Scholes PDE (above) and $ds = rs\,dt + \sigma s\,dw$ then the payoff $\phi(s(T))$ is replicated by a self-financing trading strategy with initial value (at time 0) $V(s(0),0)$, which holds $V_s(s(t),t)$ units of stock at time t.

- Analogues of the last two items for options on a forward price (in this case $V(F,t)$ solves $V_t + \frac{1}{2}\sigma^2 F^2 V_{FF} - rV = 0$ for $t < T$ and the SDE is $dF = \sigma F\,dw$).

Sections 8 and 9: Interest-based derivatives

Concepts:

- Be able to explain how a swap can be viewed as either the difference between a fixed rate bond and a floating rate bond or as a series of forward rate agreements.

- Be able to explain how you can lock in, at time 0, $F(t,T) = B(0,T)/B(0,t)$ as the discount rate for borrowing from t to T.

Computations:

- Be able to compute the present value of a swap, or the par swap rate for a swap, given the payment dates and the relevant discount factors.

- Be able to translate between discount factors, forward rates, the prices of fixed-rate bonds, etc

- Know which version of Black's formula to use for valuing a particular instrument (e.g. a caplet, floorlet, or swaption). Be able to apply it, given appropriate information (this entails, for example, evaluating the relevant forward price).

- Be able to value an interest-based option on a tree.

Proofs:

- Be able to explain why a floating rate bond that pays the risk-free rate should always be priced at par on a coupon payment date.

Note: The material in Section 10 will not be on the exam. Also: the justification of Black's formula (which occupies a few pages of Section 9) will not be on the exam. (The *use* of Black's formula for interest-based options will however be on the exam.)

Section 11: Single-name credit

Concepts:

- Be able to explain why some investors might want to buy – and others might want to sell – credit default swaps.

- Be able to explain the similarities and differences between an asset swap and a credit default swap.

- Be able to explain the logic behind the Merton model for estimating default probabilities from market data.

- Be able to explain why we should not expect the default probabilities extracted from the prices of bonds to match the historical default probabilities of companies similar to the one under consideration.

Computations:

- Given the market prices of fixed-coupon bonds issued by a particular corporation with various maturities, and knowledge of the risk-free discount factors for each payment date, find the (risk-neutral) probability of survival to each payment date.

- Given the conditional default probabilities and appropriate risk-free discount factors, find the value of a particular credit default swap, or else calculate the par CDS spread.

- Assess whether the Merton model has been properly calibrated (by testing whether a particular choice of V_0 and σ_V are consistent with the current stock price E_0 and the current implied volatility σ_E).

Proofs:

- Be able to explain the relationship between the probability of survival to time t_i (S_i), the conditional probability of default between t_{i-1} and t_i given survival to t_{i-1} (d_i), and the probability of default between t_{i-1} and t_i (p_i). (Briefly: $S_i = S_{i-1}(1 - d_i)$ and $p_i = S_{i-1} - S_i = S_{i-1}d_i$.)

- Be able to explain the origin of each term in the formula for pricing a defaultable bond, or a credit default swap.

Note: The material in Section 12 (our discussion of multiname credit products and default correlation) will not be on the exam.