

**Derivative Securities, Fall 2007 – Homework 5.** Distributed 10/31/07, due 11/14/07.

**Comments added 12/2/07.** (1) Some of the questions in this problem set ask you to price “an option” (e.g. “an option on the average value of the stock” in 2b) without clearly specifying explicitly the nature of the option. It should always be a call with strike 90. (2) Question 4 supplies redundant, inconsistent data. Suggested fix: ignore the information on a one-year swap (the remaining data are non-redundant).

*A solution sheet to HW4 will be posted 11/8; a solution sheet to HW5 will be posted 11/29 (in view of Thanksgiving); no late HW's will be accepted once the solution sheet has been posted.*

Please note that Derivative Securities final exam will be Wednesday 12/19 at the usual class time and place. Also: this class does not meet Wed 11/21 (the Wednesday before Thanksgiving), since only Monday classes meet that day.

This problem set calls for a considerable amount of arithmetic. Students with experience in programming will probably find it easiest to do some of the problems by writing short computer programs, rather than doing all the arithmetic by hand.

1. Consider a European-style call option written on a dividend paying stock with a maturity of 3 years. Assume a current stock price of 100, strike of 90, a risk-free rate of 7%, a constant dividend rate of 5%, and a volatility of 25%. Form a binomial tree with nodes at the end of each year. Use branching probabilities of 50% up/ 50% down for simplicity of calculation. Show that the option can be valued in three different ways, arriving at the same answer in all three: (1) numerical integration, (2) solving backwards on the tree, (3) Monte Carlo simulation (giving equal weight to each of the eight possible paths). (This is very similar to the example done in the Section 8 notes posted on Blackboard, except that there are 3 time periods rather than 2.)
2. Use the tree you built in Problem 1 to find:
  - (a) The value of an American-style call option, exercisable at the end of each year, evaluated by working backwards in the tree.
  - (b) The value of an option on the average value of the stock, averaged over the three year-ends, evaluated by Monte Carlo simulation.
  - (c) The value of a barrier option that knocks out if the price at the end of either year 1 or year 2 is below 85, evaluated either by working backwards in the tree or by Monte Carlo simulation.
3. Use the tree you built in the Problem 1 to value the following options. In each case, say whether you are using Monte Carlo simulation or working backwards in the tree, and state the reason for your choice.
  - (a) The value of an option whose payoff is the maximum value of the stock over the three year-end dates.
  - (b) The value of a compound option on a 3 year call struck at 90, which will expire unless the option holder makes a payment of 20 at the end of year 2 (see Hull section 22.4).
  - (c) The value of a chooser option that, at the end of 2 years, allows the option holder to choose whether the option will be a call at a strike of 90 or a put at a strike of 90 (see Hull section 22.5).
  - (d) The value of a lookback option that pays the amount that the final stock price exceeds the minimum stock price achieved during the life of the option (see Hull section 22.8). In calculating the minimum stock price, you should include both the opening price of 100 and the final price.

4. You are given the following market quotes:

3 month LIBOR	=	4.55%
1st 3 months forward (starts in 3 months)	=	5.00%
2nd 3 months forward (starts in 6 months)	=	5.35%
3rd 3 months forward (starts in 9 months)	=	5.75%
1 year swap	=	6.25%
1.5 year swap	=	6.50%
2 year swap	=	6.70%
2.5 year swap	=	6.90%
3 year swap	=	7.00%

Using bootstrapping, derive a set of semi-annual discount rates from these inputs.

5. You are given the following discount factors:

1 year	=	.95
2 year	=	.895
3 year	=	.857
4 year	=	.795
5 year	=	.745

Derive, for years 1 through 5, the zero coupon rates, par coupon rates, and forward rates. In each case, you should assume annual compounding.

6. Suppose the LIBOR discount rates  $B(0, t)$  are given by the table below. Consider a 3-year swap whose floating payments are at the then-current LIBOR rate, and whose fixed payments are at the term rate of  $R_{\text{fix}}$  per annum.

<u>payment date <math>t_i</math></u>	<u><math>B(0, t_i)</math></u>
0.5	.9748
1.0	.9492
1.5	.9227
2.0	.8960
2.5	.8647
3.0	.8413

- Suppose  $R_{\text{fix}}$  is 6.5 percent per annum and the notional principal is 1 million dollars. What is the value of the swap?
- What is the par swap rate? In other words: what value of  $R_{\text{fix}}$  sets the value of the swap to 0?