Derivative Securities
G63.2791, Fall 2004
Mondays 7:10–9:00pm
109 WWH

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Content: An introduction to arbitrage-based pricing of derivative securities. Topics include: arbitrage; risk-neutral valuation; the log-normal hypothesis; binomial trees; the Black-Scholes formula and applications; the Black-Scholes partial differential equation; American options; one-factor interest rate models; swaps, caps, floors, swaptions, and other interest-based derivatives; credit risk and credit derivatives.

Lecture notes: Lecture notes, homework assignments, etc. will be posted on my web page in pdf format – normally within a day of when they are distributed. I'll build a fresh set of notes, homeworks, etc as we go along, but at the top of the course web page you'll find a link to the page built when I last taught the class, in Fall 2000. This fall's version will be similar, except that we'll do one or two lectures on credit risk and credit derivatives near the end of the semester.

Prerequisites: Calculus, linear algebra, and discrete probability. Concerning probability: students should be familiar with concepts such as expected value, variance, independence, conditional probability, the distribution of a random variable, the Gaussian distribution, the law of large numbers, and the central limit theorem. These topics are addressed early in most undergraduate texts on probability, for example K-L Chung and F. Aitsahlia, *Elementary probability theory : with stochastic processes and an introduction to mathematical finance* Springer 2003, on reserve in the CIMS library.

Course requirements: There will be approx 7 homework sets, one every couple of weeks. Collaboration on homework is encouraged (homeworks are not exams) but registered students must write up and turn in their solutions individually. There will be an in-class final exam.The first class is Monday Sept 13; the last class is Monday Dec 13; the final exam is Mon Dec 20.

Books: We will not follow any single book linearly. However to master the material of this course you should expect to do plenty of reading. I recommend purchasing at least these two books:

- J.C. Hull, *Options, futures and other derivative securities*, 5th edition.
- M. Baxter and A. Rennie, *Financial calculus: an intoduction to derivative pricing*, Cambridge University Press, 1996.

The NYU bookstore has ordered about 30 copies of each; you may be able to save money by buying them used. Earlier editions of Hull will be sufficient for this class, but the 5th edition has some new sections on advanced or rapidly-developing topics like credit. These two books go far beyond the scope of this course; roughly, they cover both Derivative Securities and its spring sequel Continuous Time Finance.

Here are some additional books you may wish to buy or at least consult:

- R. Jarrow and S. Turnbull, *Derivative securities*, Southwestern, 2nd edition
- M. Avellaneda and P. Laurence, *Quantitative Modeling of Derivative Securities*, CRC Press, 1999.
- P. Wilmott, S. Howison, and J. Dewynne, *The mathematics of financial derivatives - a student introduction*, Cambridge University Press, 1995
- S. Neftci, *An introduction to the mathematics of financial derivatives*, Academic Press, 2nd edition.
- S. Shreve, *Stochastic calculus for finance I: The binomial asset pricing model*, Springer-Verlag, 2004

All these books are on reserve in the CIMS library. Some brief comments: Jarrow-Turnbull has roughly the same goals as Hull. I find it clearer on some topics, though Hull is the industry standard. Wilmott-Howison-Dewynne is especially good for people with background in PDE but unfortunately it de-emphasizes risk neutral valuation. Neftci provides a good introduction to the most basic aspects of stochastic differential equations and the Ito calculus (the first edition is sufficient for this purpose). Shreve's book, hot off the presses, is a lot like the first part of Baxter-Rennie, and a lot like the first half of this course. (His *Stochastic calculus for finance II: continuous-time models* was just published; it corresponds roughly to our classes Stochastic Calculus and Continuous Time Finance.)

An FAQ about probability: Math finance students often ask me for suggestions how to enhance their knowledge of probability, for example in connection with the class Stochastic Calculus. Professor Goodman is teaching Stochastic Calculus this fall, and he'll undoubtedly provide his own reading list. But here are some suggestions of my own:

(a) CALCULUS-BASED PROBABILITY. This material (at the level usually taught to upper-level math majors) is a prerequisite for Stochastic Calculus. There are many good texts. The one by K-L Chung and F. Aitsahlia (*Elementary probability theory with stochastic processes and an introduction to mathematical finance*, Springer-Verlag, 2003) has the advantage of including some material at the end that overlaps with Derivative Securities. Earlier editions (by Chung alone) cover the probability without the finance; they're just as useful.

(b) MORE ADVANCED PROBABILITY BOOKS. Past students have found it useful to read parts of the book by Z. Brzezniak and T. Zastawniak (*Basic stochastic processes : a course through exercises*, Springer-Verlag, 1999) and/or the one by S. Resnick ((*A probability path*, Springer-Verlag, 1999). The former includes a lot of material on

Markov chains; the latter includes an introduction to measure theory as it interfaces with probability. Neither book covers stochastic calculus or its applications to finance.

(c) STOCHASTIC CALCULUS. Students with relatively little background should certainly look at S. Neftci's book (*An introduction to the mathematics of financial derivatives*, Academic Press), with the warning that it only scratches the surface. You might also find T. Mikosch's book (*Elementary stochastic calculus with finance in view*) helpful, but be warned that it's more a list of facts than an explanation of them. Students with sufficient background find J.M. Steele's book a pleasure to read (*Stochastic calculus and financial applications*, Springer-Verlag, 2001). The newest addition to the list is Volume II of S. Shreve's book (*Stochastic calculus for finance II: continuous-time models*, Springer-Verlag, 2004). Its first half corresponds to our Stochastic Calculus course; its second half is similar to our Continuous Time Finance course.

All the probability books suggested above are on reserve in the CIMS library (except Mikosch, which is on order; it will go on reserve when it arrives).