(1) Consider the one-period trinomial model with
   - asset 1 = risk-free, interest rate \( r > 0 \)
   - asset 2 = risky, initial unit price \( s_0 \), final unit prices \( s_{0d}, s_0, s_{0u} \)

with \( d < 1 < u \). Assume that \( d < e^{rT} < u \) so the market admits no arbitrage. You want to buy a call option on the risky asset with strike price \( K \). What are the largest and smallest prices you should consider paying for it, based on considerations of arbitrage?

(2) Consider a forward contract on a non-dividend-paying stock, with strike price \( K \) and maturity \( T \). Its value at time 0 is \( s_0 - Ke^{-rT} \), where \( r \) is the risk-free rate (assumed constant) and \( s_0 \) is the stock price at time 0. We explained this in Section 1, using the standard “cash-and-carry” argument. Explain how that argument can be formalized using a one-period model with two assets and \( M \) states.

(3) Consider the following one-period market with 3 assets and 4 states:
   - Asset 1 is a riskless bond, paying no interest.
   - Asset 2 is a stock with initial price 1 dollar/share; its possible final prices are \( d \) and \( u \), with \( d < 1 < u \).
   - Asset 3 is another stock with initial price 1 dollar/share and possible final prices \( d \) and \( u \) (same \( d \) and \( u \)).
   - To keep the arithmetic simple, let’s assume that \( u = 1 + \epsilon \) and \( d = 1 - \epsilon \) for some \( \epsilon > 0 \). To avoid confusion, let’s number the states: 1 = both stocks go up; 2 = asset 2 goes up, asset 3 goes down; 3 = asset 2 goes down, asset 3 goes up; 4 = both stocks go down.

(a) What is the associated cash-flow matrix \( D_{1\alpha} \)?

(b) Find all the risk-neutral probabilities.

(c) Consider the contingent claim with payoff \( f = (f_1, f_2, f_3, f_4) \). What are the smallest and largest prices for \( f \) permitted by arbitrage considerations? (Let’s call these \( V_{-}(f) \) and \( V_{+}(f) \)).

(d) Does \( f_\alpha \geq 0 \) for all \( \alpha \) and \( V_{-}(f) = 0 \) imply \( f = 0? \) Explain.

(e) Which \( f \)'s are replicatable?

(4) It is said that a London betmaker gave the following odds on the 1996 US Presidential election: 6-1 in favor of Clinton, 7-2 against Dole, 50-1 against Perot. Interpret this to mean that the betmaker was willing to take only three types of bets – that Clinton would win, that Dole would win, and that Perot would win – and
• 6 dollars bet on Clinton would return 7 if he won, 0 if not;
• 2 dollars bet on Dole would return 9 if he won, 0 if not;
• 1 dollar bet on Perot would return 51 if he won, 0 if not.

Interpret this as a one-period market with three assets: a 1-dollar bet on Clinton, a 1-dollar bet on Dole, and a 1-dollar bet on Perot. What are the associated risk-neutral probabilities? How much was the betmaker taking of every dollar bet? Explain. (This problem is adapted from Marco Avellaneda’s notes.)