Continuous Time Finance Semester Review, Spring 2004

The final exam will be Wednesday May 5, in the usual class location and timeslot. You may use two sheets of notes (any font, both sides) but no other books, notes, or materials. Expect the exam to be about 1/3 on Segment 1, 1/2 on Segment 2, and 1/6 on Segment 3. All material covered through 4/14 or on any homework is fair game; there will be no questions on stochastic volatility (the 4/21 lecture). This Semester Review is intended to help you see the "big picture" and to give you some idea what kind of questions might be on the exam. (Not everything here will be on the exam, and there may be things on the exam which are not mentioned here.)

SEGMENT 1: Continuous-time methods in the equity-based setting (Sections 1-3 and Homeworks 1-2)

Topics included: understanding pricing and hedging via the Black-Scholes PDE and via the martingale representation theorem; understanding the market price of risk and its relation to the existence and uniqueness of the risk-neutral measure; forward measures associated with risky numeraires. Applications including pricing and hedging of exchange options and quantos.

Possible exam questions: (a) Price or hedge an option similar to one on the homework. (b) What is the SDE for the dollar-to-pound exchange rate under the dollar investor's RN measure? What is it under the pound investor's RN measure? (c) Explain why all tradeables must have the same market price of risk. (d) Consider two tradeables whose prices S and N solve the SDE's .... What is the SDE for S under the forward risk neutral measure associated with N?

SEGMENT 2: Interest-based derivatives (Sections 4-8 and Homeworks 3-5)

Topics related to Vasicek-Hull-White short-rate model included: pricing and hedging via PDE's (Feynman-Kac formula); derivation and use of $P(t,T) = A(t,T)\exp[-B(t,T)r(t)]$; lognormality of $P(t,T)$; capacity to match any term structure; pricing and hedging of options on bonds via Black's formula; relevance of this for caplets and swaps; trinomial trees. Also discussed more advanced models: Heath-Jarrow-Morton and the Libor Market Model.

Possible exam questions: (a) Price or hedge an option similar to one on the homework. (b) Consider the following formula for the value of a caplet ...: explain how it amounts to a formula for the solution of a certain PDE. (c) Explain why, in the context of the Hull-White model, $P(t,T)$ is lognormal. (d) What SDE does $P(t,T)$ satisfy under the RN measure? What about under the forward risk-neutral measure? (e) When constructing a trinomial tree for the SDE $dx = -ax\,dt + \sigma\,dw$ we usually use .... What system of linear equations should be solved to find the probabilities p_u, p_m, and p_d? Why is it necessary to truncate the tree? (f) Explain the statement "HJM models usually lead to non-Markovian short-rate processes."

SEGMENT 3: Approaches to the volatility skew/smile (Sections 9-10 and Homework 6)

Topics included: relevance of “fat tails”; simple formula for implied vol when σ depends only on t; the Dupire equation; jump-diffusion models. [Stochastic vol too but it won’t be on the exam.]

Possible exam questions: (a) Like problem 2 of HW6, (b) Consider a jump diffusion model in which all jumps are positive: what would the implied vol skew/smile look like? (c) Explain why $c_{KK}(K,T) = e^{-rT}p(K,T)$.