Calculus of Variations
G63.2650 (Advanced Topics in Analysis), Fall 2009
Bob Kohn, kohn@cims.nyu.edu
Wednesdays 1:25-3:10
Revised Syllabus, 8/28/09

Prerequisites: Real Variables, and PDE.

Calendar: First class 9/9/09. No class 9/16/09. Last class 12/16/09 (Reading Day, makeup for 9/16). This will be graded as a seminar class.

Course Description: A modern introduction to the calculus of variations. Some lectures will correlate with

- J. Jost and X. Li-Jost, *Calculus of Variations*, Cambridge University Press (costs about $60 for the 2008 paperback edition)

which provides an excellent introduction in the same spirit as this class. All lectures will be supported by (handwritten) lecture notes, which will include selected exercises (recommended but not required) and suggestions where to read more.

Revised plan: The following plan differs from the one posted last summer by (i) spending more time on classical topics, and (ii) being more correlated with Jost's book.

First half-semester: some classical topics

1. The direct method of the calculus of variations
2. Convexity and duality
3. Geodesics
4. Hamilton-Jacobi equations
5. Bifurcation
6. Saddle points and the mountain pass lemma

Second half-semester: some modern topics

1. Relaxation of variational problems
2. Quasiconvexity, with links to elasticity and phase transformation
3. Γ-convergence, emphasizing basic examples such as Modica-Mortola
4. Homogenization

Library reserve: The following books are being placed on reserve in the Courant library:

- Jost and Li-Jost (see above)
- G. Buttazzo, M. Giaquinta, and S. Hildebrandt, *One Dimensional Variational Problems: an Introduction*, Oxford University Press 1998
- A. Braides, *Gamma-Convergence for Beginners*, Oxford University Press, 2002
- A. Bensoussan, G. Papanicolaou, and J-L Lions, *Asymptotic Analysis for Periodic Structures*, North Holland, 1978
- B. Dacorogna, *Direct Methods in the Calculus of Variations*, Springer-Verlag, 2nd edition, 2008