Linear Algebra Lecture 2

Prof. Richard Kleeman
January 2001

Definition 1 *Let V be a vector space and $S \subseteq V$ and non-empty. $v \in V$ is a* `linear combination of elements` *of S if there exists a finite number of $u_i \in V$ and $a_i \in F$ s.t. $v = a_1u_1 + a_2u_2 + \ldots + a_nu_n$. We say v is a* ***linear combination*** *of $u_1, u_2, \ldots, u_n$ with* ***coefficients*** *$a_1, a_2, \ldots, a_n$.*

Definition 2 *Let $S \subseteq V$ and non-empty. The* ***span*** *of S denoted by $span(S)$ is the set containing all linear combinations of the elements of S. By convention $span(\emptyset) = \{\overrightarrow{0}\}$.*

Theorem 3 *The span of any $S \subseteq V$ is a subspace of V. Moreover any subspace of V containing S must also contain $span(S)$.*

Definition 4 *A subset $S \subseteq V$ is said to* ***generate or span*** *V if $span(S) = V$. One also says that the* ***elements of S generate or span*** *V.*

Definition 5 *$S \subseteq V$ is said to be* ***linearly dependent*** *if there exist a finite number of distinct $u_i \in V$ and $a_i \in F$ (which are not all zero) such that $a_1u_1 + a_2u_2 + \ldots + a_nu_n = 0$. Any subset S that is not linearly dependent is said to be* ***linearly independent.***

Theorem 6 *Let $S_1 \subseteq S_2 \subseteq V$. If S_1 is linearly dependent then so is S_2.*

Corollary 7 *Let $S_1 \subseteq S_2 \subseteq V$. If S_2 is linearly independent then so is S_1.*