Atmospheric Dynamics

Lecture 9: The mean general circulation

1 Introduction

As a first approximation the circulation of the Earth's atmosphere can be considered to be zonally symmetric. Considerable progress in understanding has occurred by making this assumption. There are two major components to this circulation, the Hadley and Ferrel cells. The first is thermally driven and is due basically to the temperature gradient between equator and pole. The second is more complex and is due primarily to the effects of baroclinic eddies whose linear aspects were examined in the previous Lecture. As we shall see these two factors are not unconnected and it is possible to envisage a complete theory of the global circulation. This does not as yet exist in a completely satisfactory form however much progress has been made in the past three decades. We examine now the nature of both cells in turn.

2 Hadley Cell

The idea of a thermal cell caused by the meridional temperature gradient between equator and pole goes back to the 18th century and the English physicist Hadley. He was able to explain the nature of trade winds and others later the jet stream using the same conceptual framework. One question that was not really answered until this century was why the observed Hadley cell extends only to around 30°. We address this below. First however we consider the implications of a thermally direct cell on a rotating sphere.

2.1 Angular momentum considerations

Let us assume that the meridional temperature gradients set up an overturning cell with air rising at the equator, sinking further poleward and then returning near the surface as trade winds. In actuality this process is aided by the presence of moist convection and hence diabatic heating which preferentially occurs above regions of high surface temperature (and hence moisture). For our present purposes we shall ignore this complication and focus purely on the direct thermal circulation. Such a situation actually occurs in a more pure way on Mars where there is no moist convection.

Consider now parcels of air leaving the equator and heading poleward. Assume that at the height this occurs (around the tropopause) frictional forces can be ignored. The angular momentum of such a parcel is easily calculated from first principles and is

$$M = \Omega X^2 + uX = \Omega a^2 \cos^2 \phi + ua \cos \phi$$

where X is the distance between the parcel and the axis of rotation of the Earth and where ϕ is latitude and a the radius of the Earth. Assuming for simplicity that the parcel has no zonal momentum when it leaves the equator then when it reaches a latitude ϕ_0 in order to conserve angular momentum it must satisfy

$$\Omega a^2 = \Omega a^2 \cos^2 \phi_0 + u(\phi_0) a \cos \phi_0$$

or in other words it must acquire a (westerly) zonal velocity of

$$u(\phi_0) = \Omega a \sin \phi_0 \tan \phi_0 \tag{1}$$

The values here are very large. At 30° a simple calculation gives $u(\phi_0) \sim 100ms^{-1}$ and clearly as the latitude becomes greater huge values would occur. This simple parcel conservation of angular momentum is the basic cause of the subtropical jet stream. At such high velocities very significant instability process become possible (as we saw in the previous Lecture) which implies that dissipation becomes non-ignorable.

At the lower levels of the atmosphere where the return flow towards the equator occurs, strong dissipative processes connected to the boundary layer occur. Nevertheless consideration of a parcel of air starting its return journey to the equator shows that angular momentum conservation implies that it should acquire a strong easterly component as it approaches the equator. Hence the northern hemispheric northeasterly trades. The zonal component of the trades is only of order $5 - 10ms^{-1}$ because of the enhanced dissipation of the boundary layer. These agreements with the observed circulations of the Earth was a major success of the Hadley model of the general circulation.

2.2 Thermal balance considerations

The presence of the meridional temperature gradient has dynamical implications which are distinct from the angular momentum balance just discussed. Clearly then the hydrostatic relation implies that such gradients must cause gradients of pressure (or geopotential) and hence circulations. Evidently this has a different dynamical origin than the conservation of angular momentum. For small Rossby numbers, which is a fair approximation for most observed flows, the balance implied by these gradients is given by the thermal wind relation discussed in a previous Lecture. Here meridional gradients of temperature imply vertical gradients of zonal velocity. If we assume that because of dissipation, velocities near the surface are close to zero then the observed upper level strong westerly flows must be consistent with the observed poleward temperature gradients.

In order to build a model of the Hadley cell we need to take this thermal constraint into consideration as well as the angular momentum one. This is obviously of importance at high latitudes since here a simple minded Hadley cell extending from equator to pole would imply as we saw, huge zonal velocities which would be inconsistent with the thermal wind relation and the roughly constant meridional temperature gradients of the mid-latitudes. This contradiction suggests that if we ignore turbulent upper atmosphere processes, then the Hadley cell cannot possibly extend to the pole. Interestingly, as we saw in the previous Lecture, very strong vertical shears in the flow lead to baroclinic instability that tries to *reduce* the poleward temperature gradient. So their effect on a Hadley cell extending "too far" to the pole would be to reduce the thermal gradients and hence the local jet and then the extent of the cell- an interesting self correction mechanism.

Simple models illustrating these ideas were developed in the late 1970s and early 1980s by Held, Lindzen, Schneider, Hou, Farrell and others. We will now consider a model due to Held and Hou [1] which illustrates how we may meet both the thermal wind and angular momentum constraints on the flow and obtain a circulation resembling the observed one. Detailed discussion of this model is given in the book by Lindzen [2].

2.3 Held and Hou model

These investigators assumed zonal symmetry and the Bousinesq approximation in a tropospheric of depth H. In addition they assumed that the atmosphere relaxes back towards a radiative equilibrium with a characteristic time scale τ. It would be more realistic to assume a relaxation towards a radiative-convective equilibrium but we ignore this complication. Further vertical diffusive (turbulent) effects are included using a Laplacian formulation. With these assumptions the steady state equations of motion become[1]

$$\begin{aligned}
\nabla \cdot \overrightarrow{u} &= 0 \qquad (2)\\
\nabla \cdot (\overrightarrow{u} u) - fv - \frac{uv\tan\phi}{a} &= \frac{\partial}{\partial z}\left(\kappa \frac{\partial u}{\partial z}\right)\\
\nabla \cdot (\overrightarrow{u} v) + fu + \frac{u^2\tan\phi}{a} &= -\frac{1}{a}\frac{\partial P}{\partial \phi} + \frac{\partial}{\partial z}\left(\kappa \frac{\partial v}{\partial z}\right)\\
\nabla \cdot (\overrightarrow{u}\Theta) &= \frac{\partial}{\partial z}\left(\kappa \frac{\partial \Theta}{\partial z}\right) - \frac{(\Theta - \Theta_E)}{\tau}\\
\frac{\partial P}{\partial z} &= g\frac{\Theta}{\Theta_0}
\end{aligned}$$

where $P = \frac{p}{\rho_0}$ and Θ is the temperature and Θ_E is the radiative equilibrium temperature which is assumed to fall in a prescribed way towards the pole and towards the surface:

$$\frac{\Theta_E(\phi, z)}{\Theta_0} = 1 - \Delta_H \left(\frac{1}{3} + \frac{2}{3}P_2(\sin\phi)\right) + \Delta_V\left(z - \frac{H}{2}\right)$$

Here $P_2(x) \equiv \frac{1}{2}\left(3x^2 - 1\right)$ and $\Theta_0 = \Theta_E(0, \frac{H}{2})$. The parameter Δ_H is the fractional drop in boyancy between equator and pole while Δ_V is the fractional

[1] The radius of the Earth is a and we retain extra spherical coordinate terms in the momentum equations in order to allow for a "thermal wind" balance at the equator. The essential derivation of the equations can be found on pp101-102 of [3].

drop of the same quantity between the tropopause and the surface. Zero momentum and heat fluxes are assumed at the tropopause which has height H. At the surface linear stress laws of the form

$$\nu \frac{\partial u}{\partial z} = Cu$$
$$\nu \frac{\partial v}{\partial z} = Cv$$

are assumed (heat flux is set to zero) and vertical velocities are set to zero at both vertical boundaries. The solutions are assumed symmetric about the equator implying that $v = 0$ at this point.

The inviscid solution of these equations with no meridional circulation is an interesting one. It must be in radiative equilibrium (i.e. we have $\Theta = \Theta_E$) and we easily derive an equation for the non-zero zonal velocity u_E:

$$\frac{\partial}{\partial z}\left(f u_E + \frac{u_E^2 \tan\phi}{a} \right) = -\frac{g}{a\Theta_0}\frac{\partial \Theta_E}{\partial \phi}$$

This is a thermal wind like balance to the radiative equilibrium potential temperature. If we assume zero velocity at the surface due to the drag there then we can easily integrate this equation and derive an expression for the zonal velocity

$$\frac{u_E}{\Omega a} = \left[\left(1 + 2R\frac{z}{H} \right)^{1/2} - 1 \right] \cos\phi$$

with the parameter

$$R \equiv \frac{gH\Delta_H}{(\Omega a)^2} \sim .226$$

for reasonable values of the parameters. Note the decrease with latitude.

It may be shown either by angular momentum considerations (see the book by Lindzen) or by direct consideration of the numerical solutions that this inviscid non-meridional solution does not hold even approximately in the tropics *because of the presence of dissipation.* In the second equation from (2) the dissipation term becomes neccessarily very large relative to the Coriolis terms violating the basis of inviscid solutions. The inviscid solution does however hold approximately outside the tropics (the Coriolis term is much larger) when the meridional (Hadley) circulation vanishes providing naturally that the dissipation is small. The properties of the solution of the equations (2) can be derived in a limit of small dissipation from a number of basic (very plausible) assumptions:

1. The upper branch of the Hadley cell conserves angular momentum.
2. The thermal wind balance holds i.e. hydrostatic and geostrophic balance.
3. Surface winds are small compared to those aloft (due to surface drag).
4. Thermal diffusion is small in the potential temperature equation.

Assumption 1 combined with 3 means that the tropopause winds in the Hadley cell must satisfy equation (1) since parcels of air lifted from the surface at the equator must have small velocities and then conserve angular momentum as they proceed poleward. The geostrophic balance can be written

$$fu + \frac{u^2 \tan\phi}{a} = -\frac{1}{a}\frac{\partial P}{\partial \phi}$$

If this is applied at the surface and the tropopause and the results subtracted and then combined with the hydrostatic relation we obtain

$$f\left(u(H) - u(0)\right) + \frac{\tan\phi}{a}\left(u^2(H) - u^2(0)\right) = \frac{-gH}{a\Theta_0}\frac{\partial\overline{\Theta}}{\partial\phi} \tag{3}$$

where $\overline{\Theta}$ is the mean (with respect to a vertical integral) tropospheric temperature. This equation is a variant of the thermal wind relation discussed above. If we assume that surface winds are small and use the conservation of angular momentum equation (1) to obtain $u(H)$ then we have

$$2\Omega\sin\phi\Omega a\sin\phi\tan\phi + \frac{\Omega^2 a^2 \tan^3\phi\sin^2\phi}{a} = \frac{-gH}{a\Theta_0}\frac{\partial\overline{\Theta}}{\partial\phi}$$

This can be integrated directly with respect to latitude ϕ to obtain

$$\frac{\overline{\Theta}(0) - \overline{\Theta}(\phi)}{\Theta_0} = \frac{\Delta_H}{2R}\sin^2\phi\tan^2\phi \tag{4}$$

This shows that the constraints imposed serve to determine the meridional variation of the mean temperature. The extent of the Hadley cell (i.e. where $v \neq 0$) can now be deduced along with the mean equatorial temperature by imposing boundary conditions for it at the edge of the cell and by requiring that the diabatic heating forcing in the temperature equation of (2) should integrate out over the domain of interest to zero: A steady state solution would not be possible otherwise since the heat fluxes out of the domain of the Hadley cell are zero because $v = 0$ outside it.

Outside the area of the Hadley cell (i.e. at high latitudes) the atmosphere must adjust to approximate radiative equilibrium by assumption 3 above since the left hand side of the potential temperature equation from (2) vanishes when there is no meridional velocity (and hence no vertical velocity). Thus for continuity of potential temperature at the edge of the cell we require

$$\overline{\Theta}(\phi_H) = \overline{\Theta}_E(\phi_H)$$

where ϕ_H is the latitude of the cell edge.

The no net diabatic heating condition can be written as

$$\int_0^{\phi_H} (\overline{\Theta} - \overline{\Theta}_E)\cos\phi d\phi = 0$$

where the cosϕ is required since area around latitudinal circles shrinks in this way. Substitution of the temperature equation (4) into these two conditions gives us two equations for the unknowns ϕ_H and $\overline{\Theta}(0)$. The relationship between $\overline{\Theta}$ and the (mean) equilibrium is shown in Figure 1.

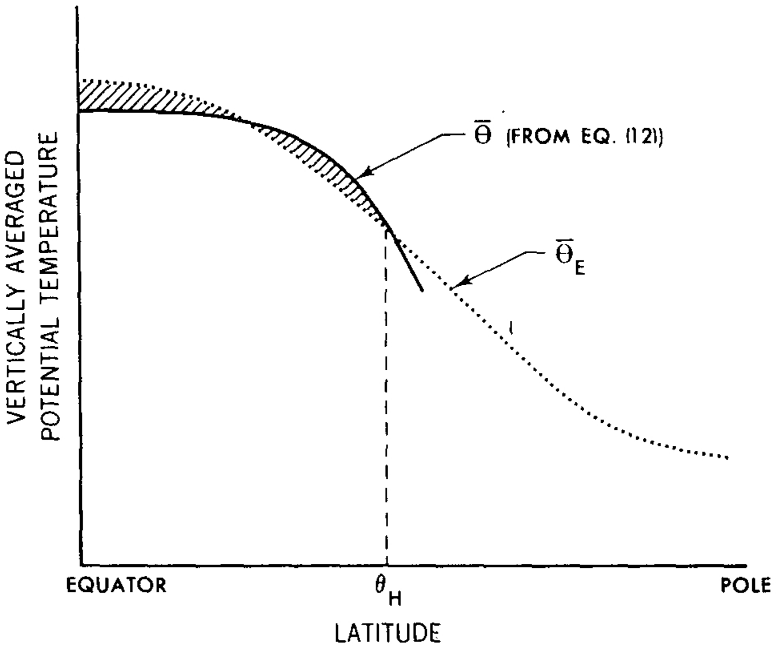

Figure 1: Vertically integrated potential temperature: Equilibrium and model values.

Approximate solutions can be obtained by assuming ϕ small (i.e. one makes standard trigonometric approximations) and yield the relation

$$\phi_H = \left(\frac{5}{3}R\right)^{1/2} \sim 35^\circ$$

Remembering the definition of the parameter R above we see that this theory predicts that the extent of the Hadley cell depends on the rotation rate of the planet as well as the relative meridional drop in the radiative equilibrium temperature between equator and pole. On Venus where the rotation rate is very small the Hadley cell may extend nearly all the way to the pole.

Notice also from the meridional gradient of the mean temperature and equation (3) that there is a discontinuity in the zonal velocity at ϕ_H (it drops suddenly) which signifies the formation of a jet. If we include significant diffusion (the parameter κ) the solutions are smoothed out somewhat and the jet strength

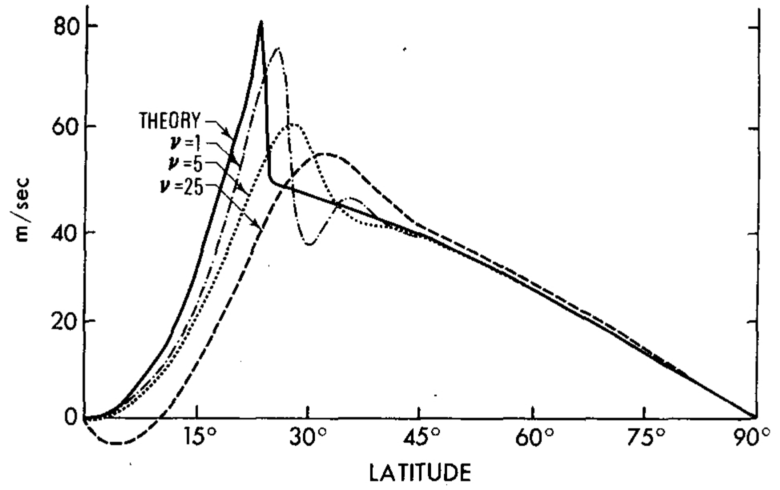

Figure 2: Tropopause values for zonal velocity for various values of the diffusion κ. The theoretical curve is that derived above in the text.

is reduced however the basic character of the solutions is not greatly altered. Figure 2 is a plot of the solutions of tropopause zonal velocity for varying κ. Figure 3 is a plot of the stream function (for meridional velocity) and zonal velocity over the full troposphere.

The solutions are remarkably realistic given the simplicity of the model we are considering. Limitations are that the Hadley cell is too weak and the jet a little too strong. The former problem is probably due to the neglect of diabatic heating caused by moist convection (which winds up the cell strength) while the latter may be due to the overly simple treatment of the effect of baroclinic eddies by a diffusive formulation. We take up this issue in another context in the next section when we consider the Ferrel cell. One very interesting feature of the solutions is that the strength of the meridional cell *increases* as the dissipation increases. Note however that the jet stream strength decreases as this happens.

3 Ferrel Cell and Eddies

3.1 Life cycle of baroclinic eddies

In the previous Lecture we looked at the linear instability properties of the mid-latitude atmosphere. We found that disturbances resembling observed synoptic storm systems and warm/cold front complexes developed as a result of vertical shears in the mean flow. This process is known as baroclinic instability and obviously the fact that many observed disturbances in the atmosphere resemble these patterns means that this process is important to the general circulation. The analysis of the previous Lecture is limited in that disturbances are assumed

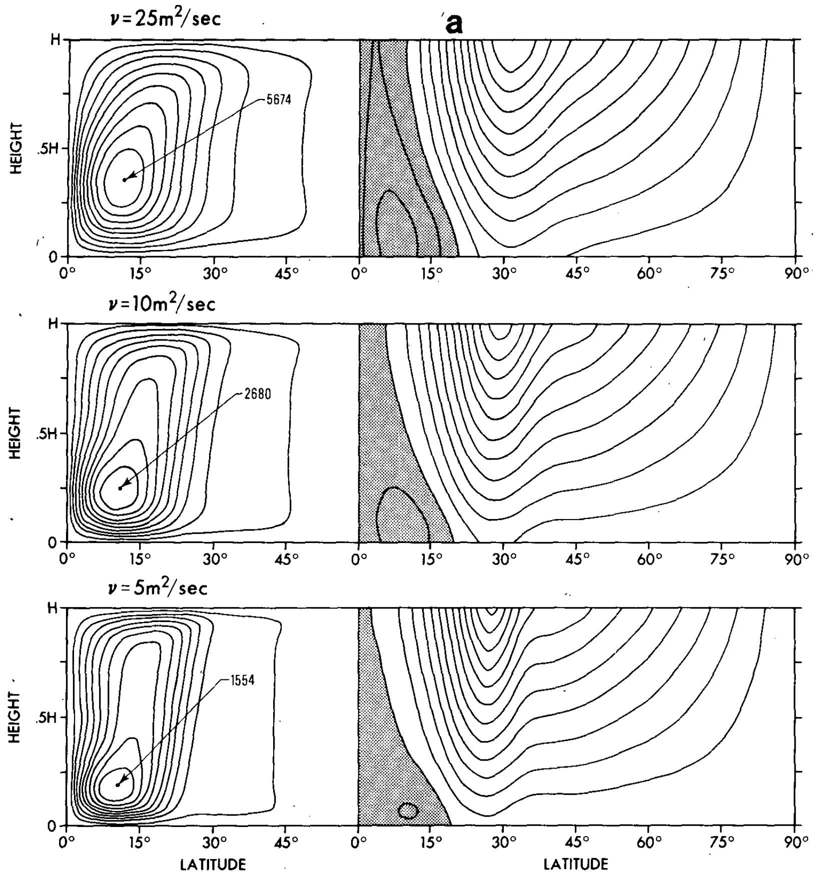

Figure 3: Meridional stream function and zonal velocity for various choices for κ the dissipation. Contour interval on the right is $5ms^{-1}$ and shaded values indicate negative zonal wind i.e. easterlies (the trade winds).

to be linear. Evidently as the amplitude of such perturbations increases to values comparable with the mean circulation, this assumption no longer holds and various non-linear phenomenon (e.g. wave breaking, shock fronts and so on) become important. Limited progress has been made in developing a general theory for these non-linear effects. Modelling studies currently offer us our best analysis tools for this complex phenomenon. We study in this section an idealized study due to Simmons and Hoskins (1980). These investigators initialized a model of the primitive equations set on a sphere with an idealized zonally constant global temperature distribution and consequent jet stream structure. This is depicted in Figure 4a.

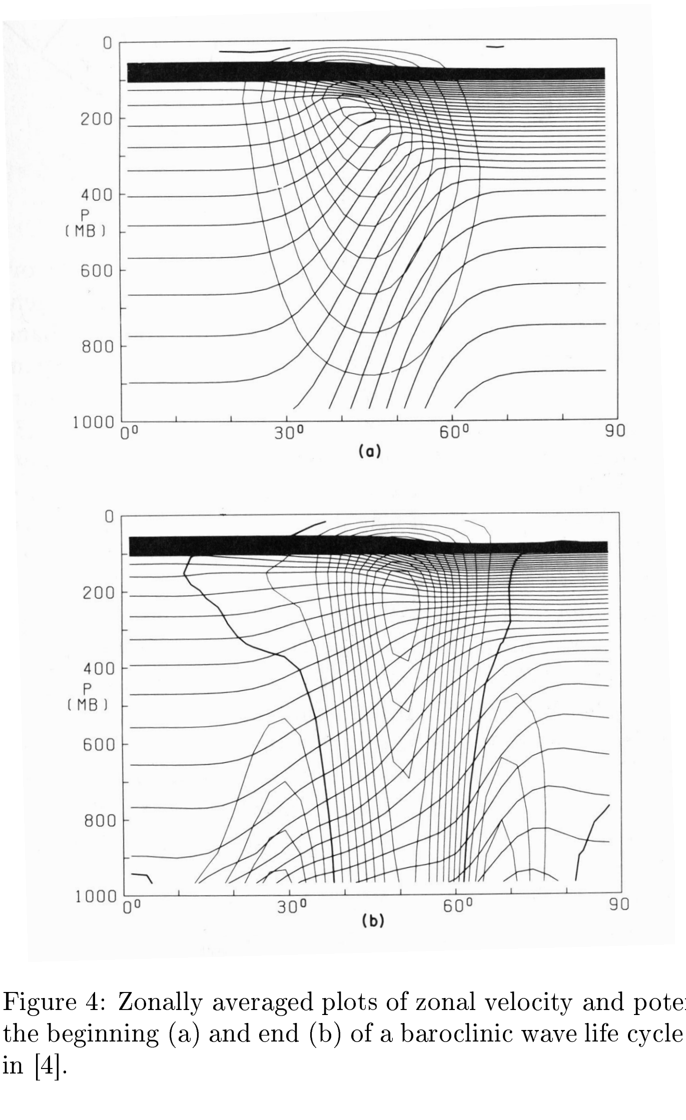

Figure 4: Zonally averaged plots of zonal velocity and potential temperature at the beginning (a) and end (b) of a baroclinic wave life cycle experiment detailed in [4].

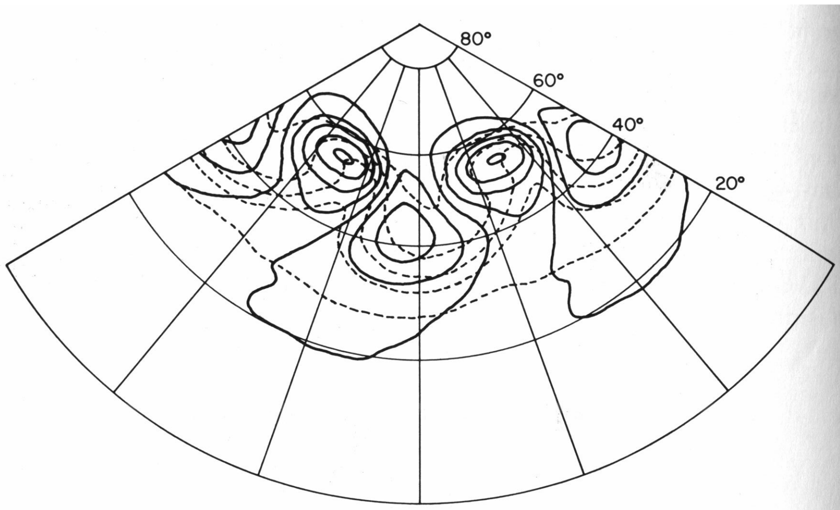

Figure 5: Horizontal structure of disturbance from previous Figure. The solid contours are pressure, the dashed are temperature.

Notice the strong mid-latitude horizontal potential temperature gradients throughout most of the troposphere and the jet stream that is in approximate thermal wind balance with these gradients. The linearized modes of this state were then calculated and the fastest growing mode (which strongly resembled the modes discussed in Lecture 8) was used to perturb the model (a quite small amplitude was used). A rapidly growing disturbance resulted whose horizontal (surface) structure is displayed in Figure 5 after 5 days.

After about this time no further growth occurs (the disturbance at this stage is comparable in magnitude to the initial mean state) and gradual decay takes place thereafter. The vertical structure after 10 days is displayed in Figure 5b.

We notice a number of things have occurred as a result of the eddy:

1. The horizontal (meridional) temperature gradient has generally been reduced.

2. Consistent with this, the vertical gradient in zonal velocity has been reduced. This has made the total state much less baroclinically unstable.

3. Westerly momentum from the jet has been transferred to the surface.

4. The jet has shifted poleward.

3.2 Eliassen-Palm Flux

Clearly as a result of this eddy there have been very significant fluxes of both zonal momentum and heat. A particularly useful way of analyzing this process is provided by the so-called *Eliassen-Palm flux* $\overrightarrow{F}$ which is defined as a vector in the $y-z$ plane with components

$$F_y = -\rho_0 \overline{u'v'}$$
$$F_z = \rho_0 f R \overline{v'T'} / \left(N^2 H\right)$$

H is a tropospheric scale height. This flux is proportional to (minus) the zonal momentum flux in the meridional direction for its meridional component and to the meridional heat flux for its vertical component. In the case that the quasi-geostrophic approximation holds the eddy adjustment is mediated by Rossby waves and one may show that the Eliassen-Palm (E-P) flux is proportional to the group velocity of these waves (see Gill, Chapter 13). The E-P flux for the model study above is depicted for times zero and eight days in Figure 6.

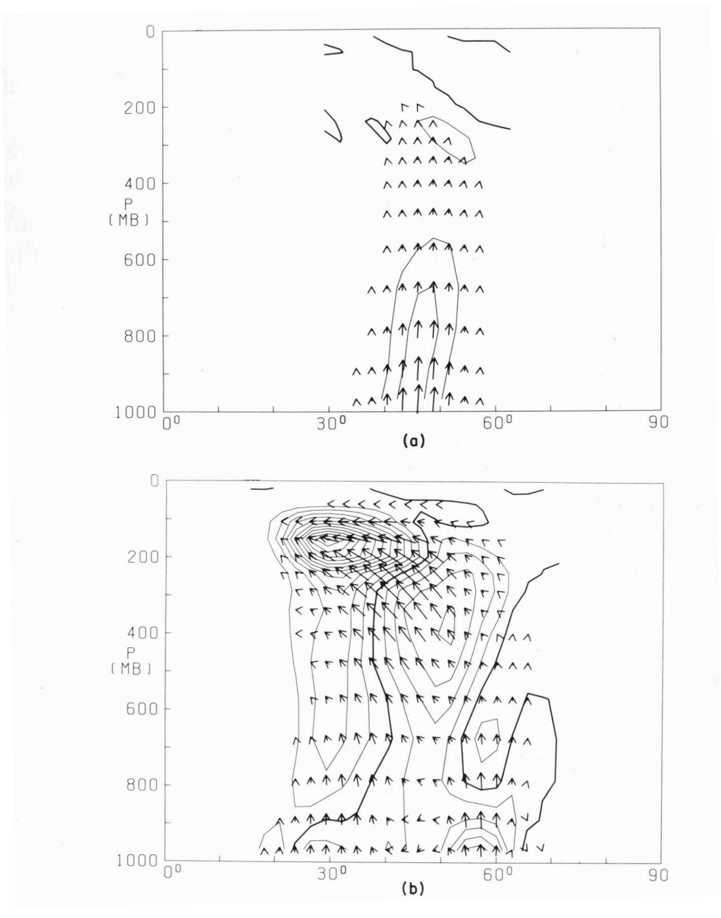

Figure 6: Eliassen-Palm flux for a baroclinic eddy. Upward arrows indicate a poleward heat flux while left directed arrows indicate a poleward zonal momentum flux. Contours indicate magnitude of the total flux.

There are a number of quite notable features here:

1. At time zero there is a strong heat flux (vertical E-P flux) operating on the meridional temperature gradient. This flux occurs primarily near the surface. Notice that there is no zonal momentum flux initially and that the Rossby wave flux is upward towards the jet. These results are also obtained in linearized solutions.

2. As the disturbance reaches maturity, the Rossby waves propagate equatorward at the level of the jet (and are actually absorbed here). This important process is associated with a strong upper level *poleward* flux of zonal momentum (c.f. the definition of the E-P flux). There is now little heat flux occurring near the surface (some is occurring at height) as the Rossby waves have propagated vertically. The reason that this process occurs within the jet is that it has velocities comparable with the group velocity of the Rossby waves causing the adjustment. Refraction and absorption of Rossby wave energy is therefore possible.

The actual E-P flux may be calculated using many years of observations of the mid-latitude Northern hemisphere and one estimate is displayed in Figure 7.

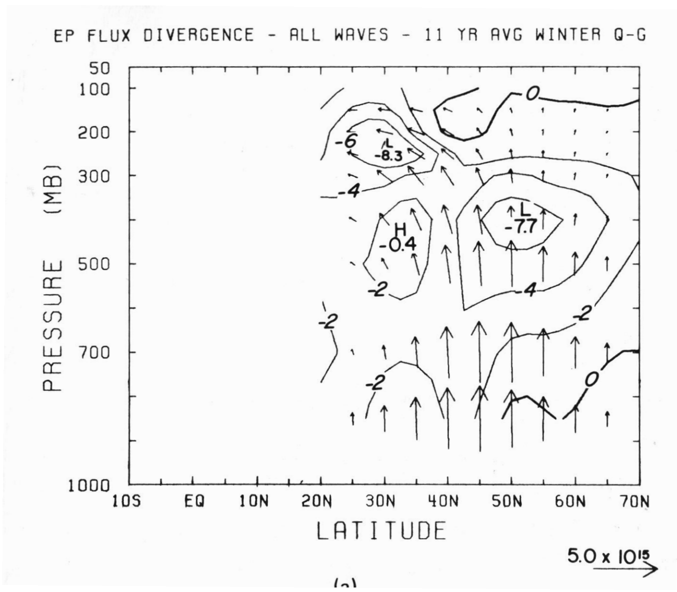

Figure 7: Observed wintertime E-P flux derived from observations (NMC analysis).

The agreement with the time average of Figure 6 is actually quite good. The northward flux of zonal momentum due to baroclinic eddies is evidently a very non-linear phenomenon and not what one would expect from a simple-minded diffusive model of the effect of eddies on the jet stream. Notice however that the poleward heat flux *can* be understood with a fairly simple linear diffusive model of the effects of eddies and consistent with this, occurs during the linear (growth) phase of the baroclinic eddy life cycle.

3.3 Effect of eddies on the mean state

Following the methodology introduced in Lecture 4 to study boundary layer eddies we split the flow into a zonal mean component $\overline{A}$ and a zonally varying component $A^{'}$ which satisfies $\overline{A^{'}} = 0$. It is easily shown now that

$$\begin{aligned}\overline{\frac{dA}{dt}} &= \frac{\overline{d}}{dt}\overline{A} + \frac{\partial}{\partial y}\left(\overline{A'v'}\right) + \frac{\partial}{\partial z}\left(\overline{A'w'}\right) \\ \frac{\overline{d}}{dt} &\equiv \frac{\partial}{\partial t} + \overline{v}\frac{\partial}{\partial y} + \overline{w}\frac{\partial}{\partial z}\end{aligned}$$

which allows us to write the zonal momentum and temperature equations as

$$\begin{aligned}\frac{\partial \overline{u}}{\partial t} - f_0\overline{v} &= -\frac{\partial}{\partial y}\left(\overline{u'v'}\right) + \overline{X} \qquad (5)\\ \frac{\partial \overline{T}}{\partial t} + N^2 H R^{-1}\overline{w} &= -\frac{\partial}{\partial y}\left(\overline{T'v'}\right) + \overline{Q}/c_p\end{aligned}$$

where $\overline{X}$ is the zonal mean stress due to small scale eddies (such as those in the boundary layer) and $\overline{Q}$ is the zonal mean diabatic heating due to radiation and convection. It is interesting now to consider long time averages of these equations where the tendency terms are very small. Such a time average should evidently cover at least the time scale of a baroclinic eddy. Clearly the eddy term in the first equation which as we saw above is large in the upper troposphere, must be balanced by a mean meridional circulation term from the left hand side (ignoring stress in this region). Likewise in the second equation, the eddy term (in the absence of diabatic heating) must be balanced by the vertical advection term i.e. there must be a mean vertical as well as meridional motion associated with the eddies. This simple-minded analysis suggests that the baroclinic eddies must induce a mean circulation. We explore this further by introducing a mean meridional mass transport stream function $\overline{\chi}$ satisfying

$$\begin{aligned}\rho_0\overline{v} &= -\frac{\partial \overline{\chi}}{\partial z} \\ \rho_0\overline{w} &= \frac{\partial \overline{\chi}}{\partial y}\end{aligned}$$

Notice that the continuity equation is automatically satisfied now. If we consider the time average circulation then the two equations from (5) can now be combined to obtain

$$\frac{\partial^2 \overline{\chi}}{\partial y^2} + \frac{f_0^2}{N^2}\rho_0\frac{\partial}{\partial z}\left(\frac{1}{\rho_0}\frac{\partial \overline{\chi}}{\partial z}\right) = \frac{\rho_0}{N^2}\left[\frac{R}{H}\frac{\partial}{\partial y}\left(\overline{Q}/c_p - \frac{\partial}{\partial y}\left(\overline{T'v'}\right)\right) - f_0\left(\frac{\partial^2\left(\overline{u'v'}\right)}{\partial z\partial y} - \frac{\partial \overline{X}}{\partial z}\right)\right]$$

If we consider a double Fourier expansion for $\overline{\chi}$ in y and z then the left hand side of this equation is proportional to $-\overline{\chi}$ and so we obtain the qualitative relation

$$\overline{\chi} \propto -\frac{\partial}{\partial y}(diabatic\ heating) + \frac{\partial^2}{\partial y^2}(large-scale\ eddy\ heatflux) + \frac{\partial^2}{\partial y \partial z}(large-scale\ eddy\ momentum\ flux) - \frac{\partial}{\partial z}(stress)$$

The first term drives the Hadley circulation as discussed above. This has $\overline{\chi} > 0$. The eddy heat flux has a (positive) maximum in the (northern) mid latitudes associated with baroclinic disturbances (see Figure 7 above) and so the second term implies a negative $\overline{\chi}$. This mid-latitude mean circulation is called the Ferrel cell. The eddy momentum flux has a maximum, as we saw above, in the upper troposphere in the subtropics. It follows that the third term contributes a positive value to $\overline{\chi}$ equatorward of the subtropics and a negative value poleward of this location i.e. in the mid-latitudes. Thus this term strengthens both the tropical Hadley circulation and the mid-latitude Ferrel circulation. The mid-latitude Ferrel Cell owes its existence purely to the presence of baroclinic eddies and in terms of magnitude is consequently considerably weaker than the Hadley Cell.

References

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