Climate Dynamics

Lecture 5: Tropical Atmospheric Circulations

1 Linear response to diabatic heating

As we saw in Lecture 2, the main driving force of the tropical circulation is provided by diabatic heating caused primarily by latent heat release which is a consequence of the large amount of precipitation that occurs in this region. We consider now the dynamical consequences of this heat engine.

In the lower layers of the tropical atmosphere adjustment time scales are generally very rapid compared to those in the ocean and are typically of the order of days to weeks. Since climatic phenomenon such as El Nino occur on much longer time scales (which are really oceanic time scales) it is useful to consider the equilibrium or steady state response of the tropical atmosphere. A useful first model of this was proposed by Gill in 1980. Here he considered a linearization about a state of rest (as we have done in previous lectures) and equilibriated the response by means of the linear dissipation terms considered in the previous lecture. In general the diabatic heating that occurs in moist convection has a very deep structure associated with it and peaks in the mid-troposphere. For this reason we can consider as a reasonable approximation that the heating projects predominantly onto the first baroclinic mode of the atmosphere which has a shallow water speed c of around $40-60ms^{-1}$. Such a mode has a vertical velocity structure peaking in mid-troposphere hence the reason for the heating projection onto this mode - see Lecture 3. Horizontal velocity and pressure perturbations are roughly the vertical derivative of this and have opposite signs in the lower and upper troposphere. The equations governing these perturbations on an equatorial $\beta-$plane can thus be written as

$$\begin{aligned} \epsilon U - \beta y V &= -h_x \\ \epsilon V + \beta y U &= -h_y \\ \epsilon h + c^2(U_x + V_y) &= -sQ \end{aligned} \qquad (1)$$

where we are equilibriating the equations with equal Newtonian cooling and Rayleigh friction. Gill assumed that ϵ was of order $(3-4days)^{-1}$ and contended that this dissipation was caused by vertical mixing processes induced by convective systems. In actual fact the Newtonian cooling term can only be ascribed to radiative cooling which has a significantly longer time scale (order two weeks). Solutions are somewhat distorted with a more realistic choice but qualitatively are similar to those we derive below. In order to easily obtain solutions Gill also assumed the long wave approximation and thus dropped the first term in the second equation of (1). Such an approximation is quite accurate for forcing with a large horizontal scale. With the long wave approximation we can derive very similar equations to those in Lecture 4 namely[1]

$$\begin{array}{rcl} (2n-1)\epsilon q^n - q_x^n & = & -s\left(Q^{n-2}/2 + (n-1)Q^n\right) \\ r^{n-1} & = & 2(n+1)q^{n+1} \\ V^{n-1} & = & \epsilon q^n + q_x^n + sQ^n \end{array} \tag{2}$$

The Kelvin mode equation is obtained by setting $n = 0$:

$$\epsilon q^0 + q_x^0 = -sQ^0 \tag{3}$$

while the first symmetric Rossby mode equation is obtained setting $n = 2$:

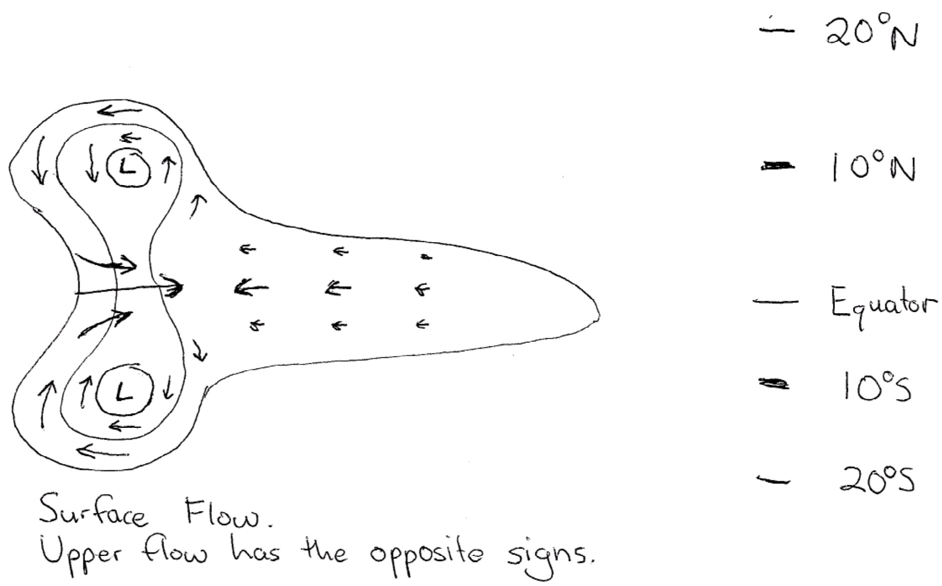
$$\begin{array}{rcl} 3\epsilon q^2 - q_x^2 & = & -s\left(Q^0/2 + Q^2\right) \\ r^0 & = & 4q^2 \\ V^1 & = & \epsilon q^2 + q_x^2 + sQ^2 \end{array} \tag{4}$$

Notice now that the forcing induces Kelvin and Rossby modes *of the same sign* unlike zonal windstress forcing which we will see in the next Lecture induces opposite signed responses. An interesting first solution may be obtained by setting

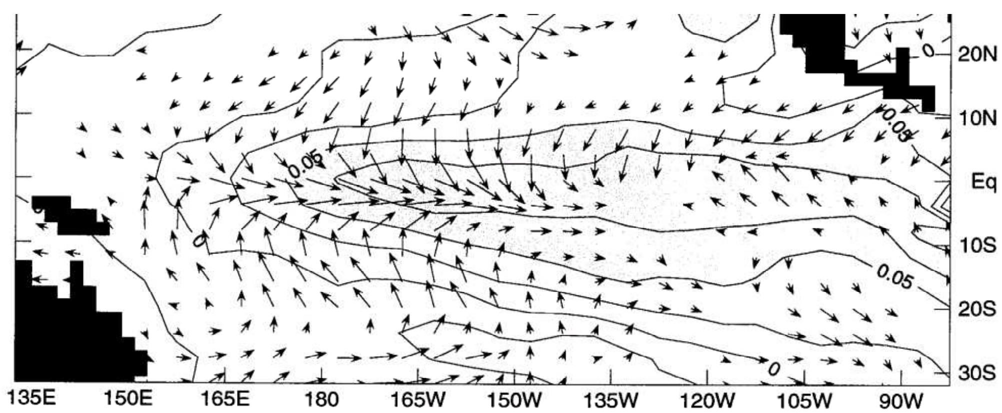
$$Q_s = AD_0(y)\sin\left(\frac{\pi x}{L}\right) \qquad 0 < x < L \tag{5}$$

and zero for other values of x. This forcing represents closely the anomalous latent heating forcing during an El Nino event which occurs around the equatorial dateline. Solutions are obtained in a straightforward fashion from equations (3) and (4) and students are urged to obtain these as an exercise. The solution is sketched in Figure 1

[1] We are non-dimensionalizing our equations as before.



while the observed circulation anomalies typically seen during an El Nino are displayed in Figure 2.

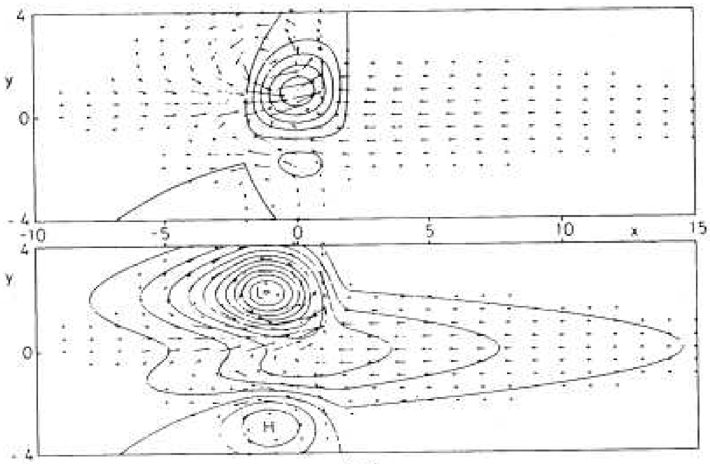


The agreement is striking given the simplicity of the model used. Notice the cyclone doublet centered to the west of the forcing and the strong zonal equatorial wind anomalies pointing down the equatorial pressure gradient. The resemblance to the oceanic solutions for zonal windstress which, as we observed is also seen in data, is very apparent. The differences here are that solutions decay away from the forcing (more rapidly on the slow moving Rossby mode side); that all perturbations are of the same sign as already noted above and finally that the atmospheric solution is much broader meridionally because the Rossby radius of deformation is around 10° for the first atmospheric baroclinic mode whereas the corresponding ocean radius is $2^\circ - 3^\circ$.

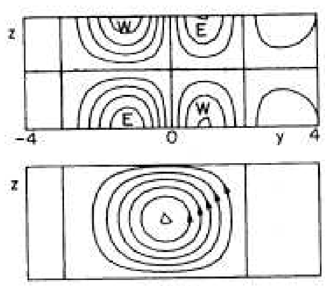
Another physically relevant solution can be obtained by adding another meridionally asymmetric part to the forcing function in (5):

$$Q_a = AD_1(y) \sin\left(\frac{\pi x}{L}\right) \qquad 0 < x < L$$

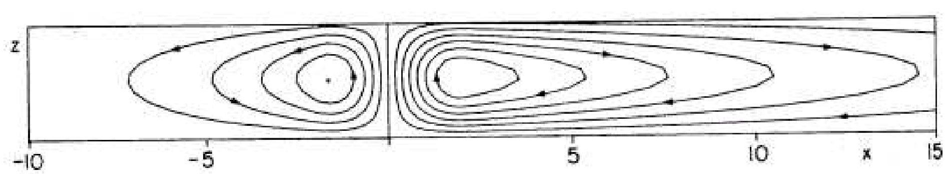
This forcing only acts on the first asymmetric Rossby mode (i.e. that with $n = 3$) and again students are urged to derive the (straightforward) complete analytical solution. The sum of forcings Q_a and Q_s give a strong heating north of the equator and a weak cooling to the south. Physically this represents the monsoonal situation seen in India with the northern heating corresponding with the heavy monsoon rains and the weak cooling with radiative cooling of the descending compressing air on the other side of the equator. The analytical solution displayed in Figure 3



(a)



(b)



(c)

resembles very well the low level flow associated with the monsoon. Notice the cross equatorial flow whose zonal component reverses sign on the equator - a characteristic feature of the monsoonal flow. Also shown is the vertical velocity which has a similar horizontal structure to the heating.

These successes in reproducing many features of the observed tropical circulation have earned the Gill model a central role in simple modeling of coupled ocean atmosphere phenomenon as we shall see in a future lecture. It is well to note however certain important limitations of this model. The largest is the uncertainty in the dissipation coefficients chosen - hardly surprising given their ad hoc nature. It is generally believed that the damping rates for momentum are appropriate for the lower troposphere where shallow cumulus convection operates nearly everywhere to provide a source of vertical mixing. In the upper troposphere however such damping is much reduced and as a result winds are significantly greater and non-linear advection terms become much more significant. Fortunately coupled ocean-atmosphere problems do not depend at all sensitively on this upper atmosphere behavior which explains the utility of the Gill model for such studies.

2 The Hadley Circulation

A fundamental feature of the Earth's[2] atmospheric circulation is that air rises in the equatorial region and descends at higher latitudes. This cell is driven by the latent heating that occurs in equatorial regions and is known as the Hadley circulation. There is an extensive body of theoretical literature on this fundamental feature which began with the English scientist Hadley in the 18th century. We shall only briefly review the tropical implications of this theory whose global character will be considered in much more detail in next semesters theoretical meteorology course.

An important analysis tool for this circulation is conservation of angular momentum: Consider a parcel of air moving in the upper troposphere from the equator to higher latitudes. Clearly for it to conserve angular momentum in this process it must obtain a large westerly velocity as it moves closer to the Earth's rotation axis at higher latitudes. A useful simple model of this can be obtained by considering zonally uniform circulations on an equatorial $\beta - plane$. Here the inviscid zonal momentum equation becomes

$$\frac{du}{dt} - \beta y v = 0$$

or

$$\frac{du}{dt} = \frac{\beta}{2}\frac{dy^2}{dt}$$

[2]This circulation has also been observed on Mars as well

which has the solution

$$u = u_0 + \frac{\beta}{2} y^2 \tag{6}$$

where u_0 is the equatorial zonal velocity of the parcel. The Hadley cell typically extends to around $30°$ so substitution of $y = 3 \times 10^6 m$ and $\beta = 2.3 \times 10^{-11} m^{-1} s^{-1}$ into this expression yields

$$u \sim u_0 + 90 m s^{-1}$$

Since equatorial upper tropospheric flows are of order $10 m s^{-1}$ there is clearly potential for the creation of very powerful westward moving jets in the upper tropospheric subtropics as a result of equatorial diabatic heating. Such jets do exist (and are called logically subtropical jetstreams) however their core velocities are around $35 m s^{-1}$ which indicates that dissipative processes must play a fundamental role in their dynamics. In a way this is obvious since equation (6) shows that the jet formation must be associated with large horizontal shears and some consideration of surface drag shows that large vertical shears must occur as well. These shears will result in respectively barotropic and baroclinic instability i.e. in turbulent mixing in respectively the horizontal and vertical. Evidently such mixing will reduce the jet strength from the inviscid upper limit. Notice however that the dissipation involved must be significantly less than that assumed in the Gill model in the previous section consistent with the comments there about the applicability of that model only to lower troposphere flow. It is also to be noted that these dissipation processes are also responsible for the Hadley flow not extending all the way to the pole as originally proposed by Hadley. One can see that if such a situation did occur then massive instability must result since the observed subtropical jet is already unstable and any hypothetical polar Hadley jet would be much stronger still. Further elementary reflection on the dissipated Hadley circulation reveals that the strength of the jetstream will depend on the strength of vertical velocity at the equator and thus on the strength of the convective heating. One would therefore expect the subtropical jet to be strongest in regions zonally where the diabatic heating is strongest. This is realized in the observations in, for example, the Indonesian/Australian sector of the Southern hemisphere.

Another interesting consequence (noticed by Hadley) of conservation of angular momentum concerns the surface flow which returns from the subtropics to the equator. This flow must acquire a significant easterly zonal component. This is the explanation for the easterly tradewinds of the tropics.

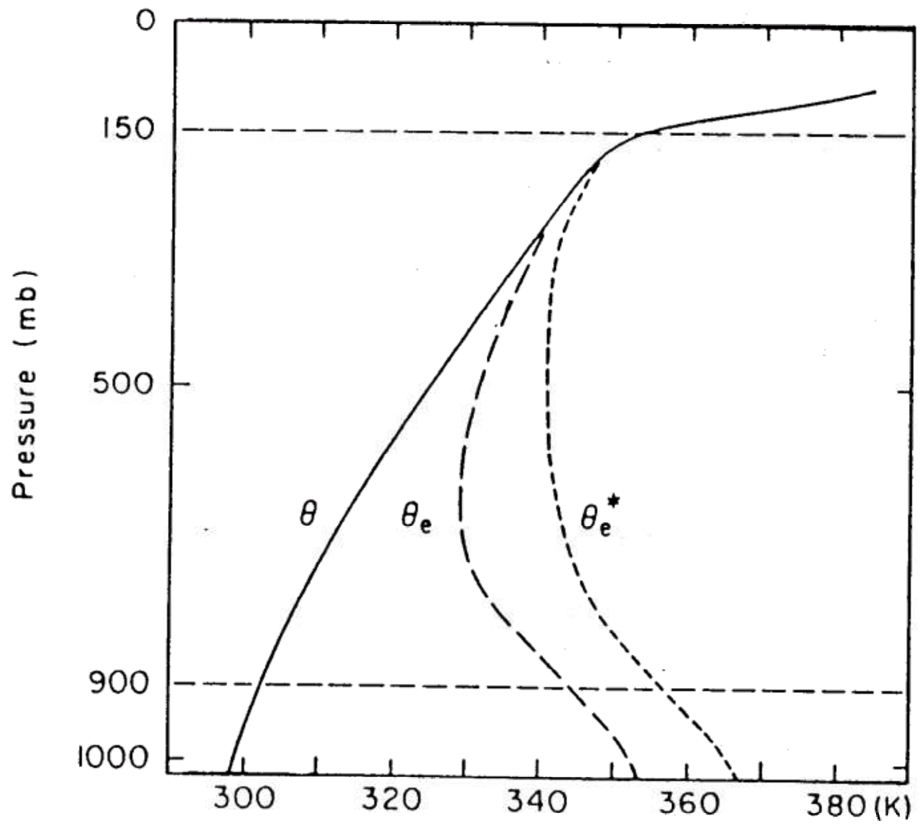
3 Tropical Convection

3.1 Basic Theory

The diabatic heating of the tropical atmosphere is a consequence of moist convection[3]. Our understanding of this turbulent process is only partial and this lack is a major cause of uncertainty in models of the Earth's climate. As we saw in Lecture 2 moist convection results when a saturated parcel of air ascends from near the surface and as it rises and cools it condenses water vapor that it can no longer hold at its cooler temperature. A large amount of this ascent occurs in large tropical thunderclouds sometimes called "hot towers" and a lot of the problem of moist convection is determining the statistical effects of such small scale towers on the large scales resolvable by climate models and conversely in determining the influence of the large scale on the statistical dynamics of the convective elements.

Useful concepts in understanding this process are those of potential temperature; saturated equivalent potential temperature and equivalent potential temperature. The first is the temperature that a parcel of air would be if it was moved adiabatically (i.e. without addition of heat) to a fixed reference pressure within the atmosphere. The second is the temperature that a *saturated* parcel of air would acquire if all its water vapor were condensed by adiabatic expansion and then it was moved to the fixed reference pressure used for potential temperature. Finally equivalent potential temperature is the temperature that *any* parcel of air would acquire if it was adiabatically expanded to saturation then until all vapor was condensed and finally compressed back to the reference pressure. Plots for all these variables in a typical tropical atmosphere are shown in Figure 4.

[3]Radiation is generally a passive process and the ubiquitous warming due to moist process implies that the tropical atmosphere is generally warmer than radiative equilibrium would suggest. This means radiation is a cooling influence in general.



Here the saturated equivalent potential temperature is for a hypothetical saturated parcel at the same height and temperature as observed.

There are a number of things to note from this diagram. Firstly the potential temperature increases with height which means that the atmosphere is stable with respect to dry convection since any parcel from below raised adiabatically will have a lower temperature than its environment and hence will be denser. This stability is a consequence of moist convection. Secondly the equivalent potential temperature is always less than the saturated equivalent potential temperature because the relative humidity of the mean atmosphere is significantly less than 100%. Thirdly if we were able to lift a parcel of air from below $900mb$ to a height at which it became saturated then it would be able to ascend until it reached near the top of the troposphere because of the shape of the saturated equivalent potential temperature profile. Notice that this is not possible for parcels higher than $900mb$. In general vertical turbulence processes which could give a parcel of air this lift are mainly confined to the same region (a rather interesting coincidence) meaning than only near surface air has the possibility of triggering convection by vertical turbulence. Of course if there is large-scale upward vertical velocity then the whole boundary layer could be lifted to its "moist unstable" level and hence wholesale moist convection could then take place. It is clear then that the properties of the near surface air are crucial to whether convection can take place.

As was mentioned in Lecture 2 moist convection is an interesting heat engine in that it can provide it own fuel: If a small convective perturbation occurs it will produce a (small) large scale upward vertical velocity this will cause larger amounts of the boundary layer to be lifted to their condensation level which will increase the heating and so on. This positive feedback loop is known as CISK (convective instability of the second kind[4]) and is thought responsible for many tropical atmospheric disturbances.

3.2 Surface controls

In reality the location of tropical convection is strongly tied to the temperature of the ocean surface which is abbreviated as SST (Sea surface temperature). Observations show that most convection in the tropics is confined to regions where $SST > 27.5°C$. Since there are vast areas of the tropical oceans where this does not hold (in particular the Southeastern Pacific) this restriction is highly significant. There are two mechanisms that cause this restriction:

3.2.1 Boundary layer thermal effects

The atmospheric boundary layer is in strong turbulent contact with the ocean surface and therefore the SST is a major factor in determining its temperature. It follows that near surface air temperature gradients will mirror to a strong degree those of the SST. A useful way of thinking of this is that the surface is heating the boundary layer via the turbulent sensible heat flux (see Lecture 2). This is a similar process to the diabatic heating considered previously and one can build a very similar linear model to the Gill model. The difference being that the vertical heating profile will be much lower and

[4]The instability of the first kind is the vertical parcel instability detailed above.

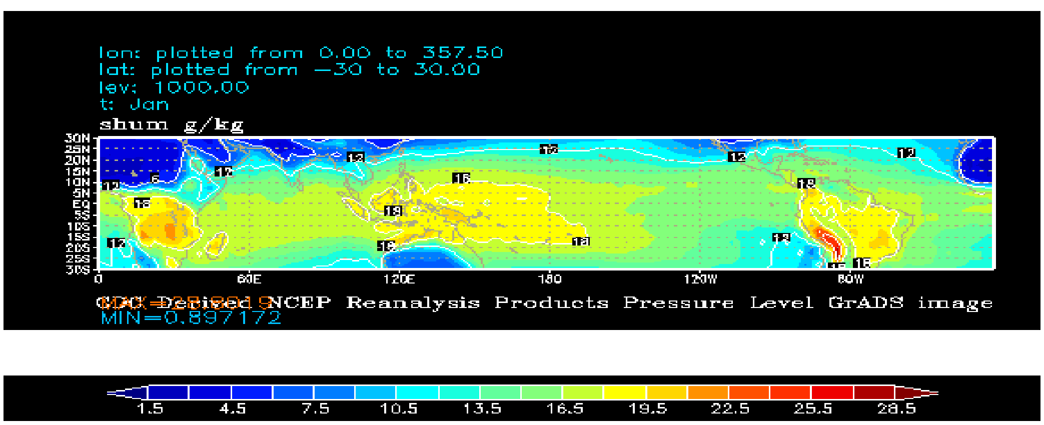
essentially confined to the boundary layer. It follows that there will be a projection onto higher order baroclinic modes and a much shallower overturning circulation will develop. The implication however is that there will be an upward vertical velocity (roughly from the surface to the top of the boundary layer) in regions where the SST is highest and conversely in regions where it is coolest. As we saw above this uplift is required for convection to develop so it is clear that this mechanism will favour convection in regions of high SST and inhibit it in low SST regions.

3.2.2 Variations in near surface humidity

The flux of moisture into the boundary layer of the atmosphere from the ocean surface is given by

$$E = \rho_a c_E W (q_{sat}(SST) - q_a)$$

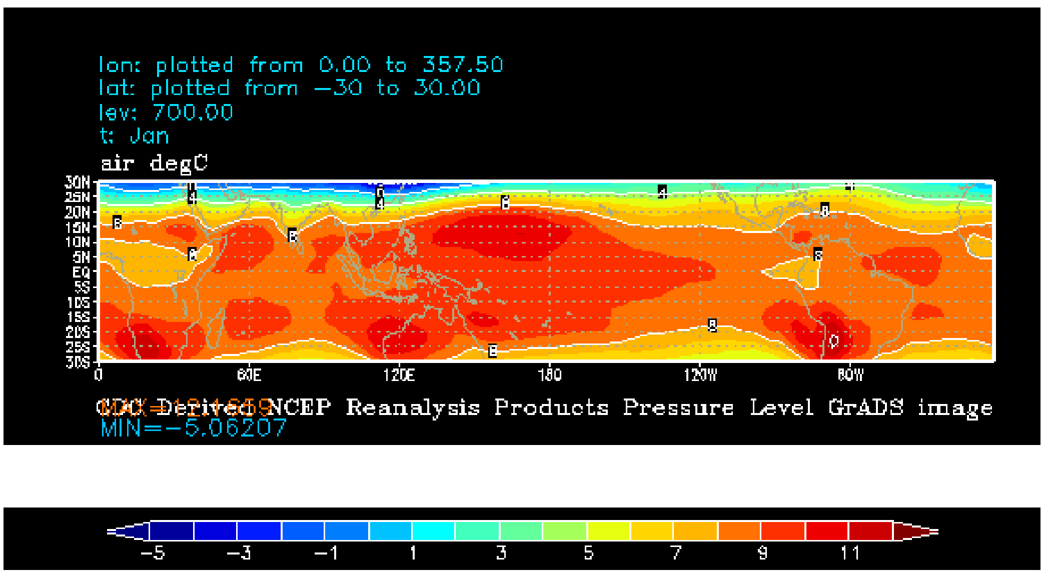
where q_a is the near surface specific humidity; $q_{sat}(SST)$ is the saturated specific humidity for air at the sea surface temperature (SST); W is the windspeed and c_E is a roughly constant exchange coefficient. This equation implies that the near surface humidity is strongly tied to SST.[5] That this is the case can be seen in Figure 5



[5]The function $q_{sat}(SST)$ is a monotonic function which increases with SST somewhat faster than a linear function. The relation is known as the Clausius Clapeyron relation.

which is a plot of mean specific humidity for January. This strong link between lower tropospheric humidity and SST has major implications for convection.

To begin with it implies that the near surface equivalent potential temperature must also be so tied. On the other hand the temperature of air higher up, which in general is very dry, does not show such a close link with SST. This is shown in Figure 6



which displays mean January temperature at $700mb$. Noteworthy is the reasonably uniform values in the deep tropics. The reason for this uniformity is not all that well understood but is undoubtedly related to the heat redistributive effects of the overturning cells caused by convection[6]. The combined effect of these two climatological features is that convection is much easier to induce in higher SST regions since the saturated equivalent potential tem-

[6] Air masses above the boundary layer have their origin mainly in the convective updrafts since direct turbulent contact with the surface is weak.

perature at levels above the boundary layer is uniform spatially whereas the equivalent potential temperature of near surface air increases markedly with increasing SST .

The above effect shows that convective instability of the first kind increases with SST. The second kind (CISK) also increases for the following reasons: If we combine the steady state tracer and mass continuity equations (see Lectures 1 and 2) we obtain

$$\nabla.\left(\overrightarrow{u}q\right)+\partial\left(wq\right)/\partial z=-P+\partial\left(F_q\right)/\partial z$$

where $\overrightarrow{u}$ is the horizontal velocity; F_q is the upward turbulent flux of moisture and P is the precipitation rate. If we integrate these equation from the surface to the top of the atmosphere we obtain

$$\int_0^\infty Pdz=-\int_0^\infty\nabla.\left(\overrightarrow{u}q\right)dz+E \tag{7}$$

where E is the surface evaporation rate. This relation shows that a given low level convergence anomaly will induce a precipitation anomaly which will be greater if the near surface humidity is larger. Thus the positive feedback process mentioned previously (the concept of a self fueling heat engine) will be much more effective when the near surface humidity is greater. This of course occurs when the SST is higher.

The combined effects of both these mechanisms mean that tropical convection is almost universally restricted in the deep tropics to regions for which $SST>27.5°C$. Such a cutoff is not an absolute limit in the sense that it's real significance is that it is roughly $2.5°C$ less than the maximum of large scale global SST. If this maximum temperature were to increase due to global warming (for example) then one would expect physically that the convective cutoff temperature would also increase.

3.3 Convective instabilities

Observations of the tropical atmosphere show a number of interesting large scale coherent convective disturbances. An example is the so-called Madden Julian Oscillation (MJO) which is an eastward propagating disturbance with a zonal scale of several thousand kilometers and a phase velocity of roughly $4ms^{-1}$. Another example is the so-called easterly waves which propagate westward along the ITCZ most notably off the African coast. Finally the most destructive coherent structure is, of course, the hurricane.

There are a number of positive feedback processes that drive these convective instabilities. The most studied in the literature is CISK which we briefly explore here: The precipitation in convective systems generally has a reasonably fixed vertical structure. This allows us to write the potential temperature equation (see Lecture 1 and equation (7)) as

$$\frac{c_p T}{\theta}\frac{d\theta}{dt} = R(z)\int_0^\infty P dz = R(z)\left(-\int_0^\infty \nabla.\left(\overrightarrow{u} q\right) dz + E\right)$$

If we linearize this equation about a background state appropriate for convection ($\overrightarrow{u} = 0$ and $\nabla q = 0$) then we obtain

$$\begin{aligned} \theta_t + w\overline{\theta}_z &= \tfrac{\overline{\theta}}{c_p\overline{T}} R(z)\left(-\textstyle\int_0^\infty q\nabla.\left(\overrightarrow{u}\right) dz + E\right) \\ &= \tfrac{\overline{\theta}}{c_p\overline{T}} R(z)\left(\textstyle\int_0^\infty q w_z dz + E\right) \\ &\approx \tfrac{\overline{\theta}}{c_p\overline{T}} R(z)\left(G(z) q_a(x,y) w + E\right) \end{aligned} \tag{8}$$

where we are using approximate incompressibility; integration by parts and assuming that q and w have approximately fixed vertical structures. Variable q_a is the near surface humidity. This equation tells us that convective heating is in general balanced by vertical advection of potential temperature (recall from Figure 4 that this increases with height). Now for given vertical profiles of w, q and $\overline{\theta}$ it may be that for some z

$$\overline{\theta}_z < \frac{\overline{\theta}}{c_p\overline{T}} R(z) G(z) q_a(x,y)$$

This could occur, for example, if q_a was large enough due to high SST. In this situation θ (and hence other dynamical variables) will grow without linear bound. This is the mathematical expression of CISK. In general nonlinearity will limit this growth as the total θ_z will increase due to the latent heating. Many models of this process exist which make certain assumptions about the vertical structure functions in equation (8) and they are able to explain many of the features of the convective instabilities mentioned above.