Atmospheric Dynamics

Lecture 4: Planetary Boundary Layer

1 Turbulence Generation

The surface of the Earth at the bottom of the atmosphere implies that fluid velocities near there are reduced by molecular diffusion to very small values. Such a reduction implies very strong vertical wind shears close to the surface. Shears in a fluid are well known as a major source of turbulence. In addition the surface temperature is subject to quite different controls from that of the atmosphere which often implies very strong vertical temperature gradients. The latter effect can generate convective overturning as discussed in Lecture 2 and operates whenever the surface has a higher temperature (during the day for example). Convective overturning is fundamentally a turbulent process as well. As a result of the above two processes turbulent eddies develop with scales ranging from millimeters up to a kilometer or so. They usually are confined to the first kilometer above the surface. This turbulent layer is called the planetary boundary layer and to a reasonable approximation has a uniform density due to the mixing effects of the turbulence.

2 Large scale, Boussinesq and geostrophic approximations in the mid-latitudes

The first two were mentioned briefly in the first lecture. They are appropriate for the planetary boundary layer flow and a full and detailed derivation may be found in Holton in Chapter 2. Here we sketch the important elements for our particular application. Firstly when a large scale (or synoptic) flow in the mid-latitudes is considered the Coriolis terms in the momentum equations simplify considerably. In addition the real atmosphere is characterized by much greater variations in density in the vertical direction rather than in the horizontal. This is basically a consequence of the dominance of gravitation in the momentum equations. If we write the density then as

$$\rho(x,y,z) = \bar{\rho}_0(z) + \rho'(x,y,z)$$

with

$$\bar{\rho}_0(z) \equiv \int \int \rho(x,y,z)dxdy$$

then for atmospheric flows over most of the troposphere $\rho' \ll \rho$. If a scale analysis of the continuity equation is then performed for large scale flows (see Holton again) then one can deduce the simpler form appropriate to large scale atmospheric flows:

$$\nabla.(\bar{\rho}_0\overrightarrow{u}) = 0$$

The Boussinesq approximation consists of replacing the density in all equations aside from the vertical momentum equation with a constant value ρ_m. It is appropriate in circumstances where density variations about this constant value are small which is the situation applicable to the planetary boundary layer. The horizontal momentum equations and the continuity equation reduce then to

$$\begin{aligned} \frac{du}{dt} &= -\frac{1}{\rho_m}p_x + fv \\ \frac{dv}{dt} &= -\frac{1}{\rho_m}p_y - fu \end{aligned}$$

$$\nabla.\overrightarrow{u} = 0 \tag{1}$$

Notice that the continuity equation has taken on the form appropriate for incompressible flow.

Finally if one is interested in large scale flows in the mid-latitudes commonly called synoptic flows then based on the observed spatial, wind and pressure variation scales of such flows in the interior of the atmosphere (see page 39 in Holton), one can deduce that the left hand side of the first two equations above is an order of magnitude less than both terms on the right hand side. If these terms are dropped then we have an approximate flow known as the *geostrophic flow*

$$\begin{gathered} \frac{1}{\rho_m}p_x = fv_g \\ -\frac{1}{\rho_m}p_y = fu_g \end{gathered}$$

Notice that such a flow does not involve time derivatives so is non-dynamic. This is referred to in meteorological parlance as a diagnostic as opposed to prognostic flow. In the next lecture we will consider a higher order prognostic version of this flow called the quasi-geostrophic approximation.

3 Reynolds averaging

One approach to understanding the effects of the turbulent eddies in the boundary layer involves the separation of the flow into a fast and slow component with respect to time. The former can be viewed as the response due to the turbulent eddies while the latter the slowly varying flow. Symbolically one separates the flow as

$$u = \bar{u} + u'$$

where the over-bar represents a long term temporal average and the prime deviations from this. By definition one assumes that the time average over the selected time interval of the deviation (or fluctuation) field is zero:

$$\overline{u'} = 0$$

It is trivial to verify for two arbitrary variables that

$$\overline{ab} = \bar{a}\bar{b} + \overline{a'b'} \tag{2}$$

Now consider the horizontal momentum equations for the boundary layer and substitute the above decomposition into both equations and perform an average with respect to time. Use of the equations (1) and (2) then results in the equations:

$$\frac{\bar{d}\bar{u}}{dt} = -\frac{1}{\rho_m}\frac{\partial\bar{p}}{\partial x} + f\bar{v} - \left[\frac{\partial(\overline{u'u'})}{\partial x} + \frac{\partial(\overline{u'v'})}{\partial y} + \frac{\partial(\overline{u'w'})}{\partial z}\right]$$

$$\frac{\bar{d}\bar{v}}{dt} = -\frac{1}{\rho_m}\frac{\partial\bar{p}}{\partial y} - f\bar{u} - \left[\frac{\partial(\overline{u'v'})}{\partial x} + \frac{\partial(\overline{v'v'})}{\partial y} + \frac{\partial(\overline{v'w'})}{\partial z}\right]$$

where the over bar on the total time derivative means

$$\frac{\bar{d}}{dt} \equiv \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{v}\frac{\partial}{\partial y} + \overline{w}\frac{\partial}{\partial z}$$

The square bracketed terms on the right hand side of the equations above represents the influence of the turbulent eddies on the mean flow and are clearly the divergence of various velocity flux terms. When divided by the mean density there are nine such terms for velocity (there are three more that appear in the unlisted vertical momentum equation) and they are called collectively the Reynolds stress tensor. Using sensitive measuring devices with high temporal resolution they can actually be measured. Of course the Reynolds decomposition does not help us solve the equations since we need to know some relation between the Reynolds stresses and the mean fields of the system. This problem is called the closure problem and is common to situations involving turbulence. There are similar terms in the temperature equation which represent the flux of heat within the system.

In the event that we are dealing with a surface that is uniform (often a reasonable assumption) then the turbulence can be considered to be horizontally homogeneous and the above equations reduce to

$$\begin{aligned} \frac{\bar{d}\bar{u}}{dt} &= -\frac{1}{\rho_m}\frac{\partial\bar{p}}{\partial x} + f\bar{v} - \frac{\partial(\overline{u'w'})}{\partial z} \\ \frac{\bar{d}\bar{v}}{dt} &= -\frac{1}{\rho_m}\frac{\partial\bar{p}}{\partial y} - f\bar{u} - \frac{\partial(\overline{v'w'})}{\partial z} \end{aligned} \tag{3}$$

Notice the similarity of this set of equations to the equations discussed in the final section of Lecture 2. The vector

$$\overrightarrow{X} \equiv \rho_m(\overline{u'w'}, \overline{v'w'})$$

is commonly called the Reynolds stress.

4 Slow large scale flow in the boundary layer

While the geostrophic approximation discussed above may be a fairly accurate one for interior large scale flows in the mid-latitudes, it is not appropriate for the boundary layer. In fact it is easily shown that there the turbulent Reynolds stresses are comparable in magnitude to the Coriolis and pressure gradient terms in equations (3). Reapplication then of the scaling argument of section 1 shows that the terms on the left hand side of equations (3) are an order of magnitude less than those on the right hand side. Thus to a good approximation we have the following equations for the slow flow in the mid-latitude boundary layer:

$$\begin{aligned} f(\bar{u} - \bar{u}_g) - \frac{\partial(\overline{u'w'})}{\partial z} &= 0 \\ f(\bar{v} - \bar{v}_g) + \frac{\partial(\overline{v'w'})}{\partial z} &= 0 \end{aligned} \tag{4}$$

It is instructive to integrate these equations over the depth of the boundary layer. At the top of this layer where the turbulent eddies are small the Reynolds stress terms also become rather small. On the other hand at the surface the stress is just given by the empirical bulk relations discussed in Lecture 2. Thus the integrated mean flow is given by

$$fV = \frac{1}{\rho_m}\frac{\partial P}{\partial x} - X_s \tag{5}$$

$$fU = -\frac{1}{\rho_m}\frac{\partial P}{\partial y} + Y_s \tag{6}$$

where the capital letters denote the slow integrated boundary layer flow and the surface stress is (X_s, Y_s). Notice the importance of the surface stress terms in determining this flow and its deviation from a geostrophic flow.

5 Simple closures and the Ekman flow

Equations (3) or (5) and (6) are not in closed form so cannot be solved without further assumption about how the turbulence within the boundary layer depends on the slow flow. One moderately successful approach to this issue is to argue in analogy with the well understood process of molecular diffusion. In that case fluxes are assumed proportional to the local gradients of a particular quantity. This assumption regarding the turbulence is known as flux gradient theory and assumes the following form for the vertical Reynolds (pseudo) stresses:

$$\begin{aligned} \overline{u'w'} &= -K_m \overline{u}_z \\ \overline{v'w'} &= -K_m \overline{v}_z \end{aligned} \tag{7}$$

where K_m is called the eddy viscosity. This quantity, as was mentioned in Lecture 2, is usually assumed in most turbulence parameterizations to be a function of the local fluids susceptibility to instability as determined by extensive laboratory experiments. As a rule these parameterizations give reasonable results when compared with observed boundary layer flows but it is clear that they are no substitute for a full resolution of the turbulent eddies. A very active area of current research is to use very high resolution models able to resolve at least a significant fraction of the eddies and then compare results with flux gradient parameterizations.

In order to develop some understanding of the basic dynamical effects of the turbulence it is common to make the crude assumption that the eddy viscosity is constant. Such an assumption was made by the first investigator in this field V. Ekman in the early 20th century and the resulting solutions are known as the Ekman flow. Substituting the flux gradient relations (7) with constant eddy viscosity into equation (4) and for convenience dropping over-bars gives the equations.

$$K_m u_{zz} + f(v - v_g) = 0$$
$$K_m v_{zz} - f(u - u_g) = 0$$

These coupled linear second order differential equations are easily solved by standard methods. It is common to assume for simplicity that the pressure is constant in the boundary layer and hence so are the geostrophic winds. Further one can assume reasonably boundary conditions that the winds vanish at the surface and match the geostrophic winds at the top of the boundary layer. Finally to simplify the interpretation consider the case that the meridional component of the geostrophic wind vanishes (the more general case adds little). Combining the two coupled equations above results in a fourth order linear equation in z. After imposing the boundary conditions we obtain:

$$u = u_g \left(1 - \exp\left(-\gamma z\right) \cos\left(\gamma z\right)\right)$$
$$v = u_g \exp\left(-\gamma z\right) \sin\left(\gamma z\right)$$

where the vertical scaling parameter is given by

$$\gamma = \left(f/2K_m\right)^{1/2}$$

Note that we are assuming that $f > 0$ i.e. we are in the Northern Hemisphere[1]. The boundary layer height is given by $h = \pi/\gamma$. These solutions represent a spiral (the Ekman spiral) which involves the rotation between u and v as height increases. Figure 1 is a plot of these variables as a function of the angle γz.

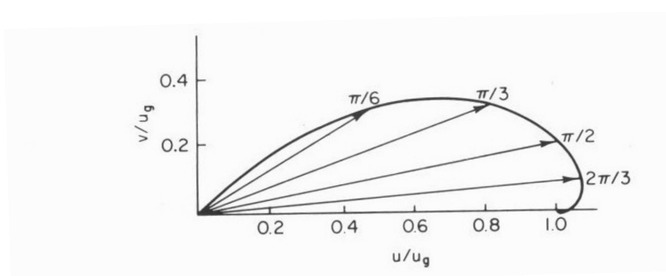

Figure 1: The Ekman spiral solution. Axes are the ratio of the solutions to the zonal geostropic wind

[1] In the Southern Hemisphere, the solution is the same except the sign of v is reversed.

In real world boundary layers spirals qualitatively similar to the Ekman solution are often observed however the marked deviations from the idealized solution is an indication of the inadequacy of the assumption of constant eddy viscosity. Plotted in Figure 2 are some real soundings from Jacksonville Florida. Also shown is a spiral resulting from a somewhat more realistic theory of K_m.

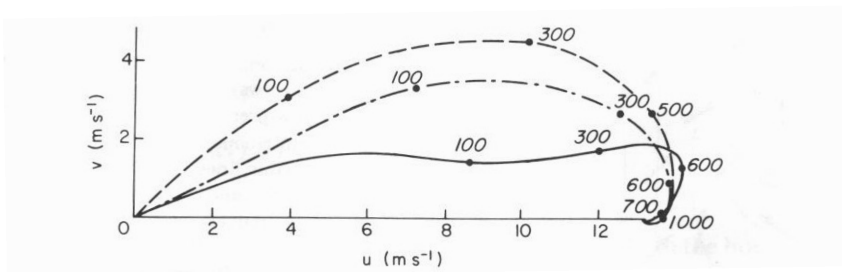

Figure 2: Observed winds in a boundary layer near Jacksonville FL. Also plotted are the Ekman spiral (upper curve) and a more realistic formulation of K_m.

6 Ekman pumping

The presence of a turbulent rotating boundary layer implies that there often is a significant vertical velocity at the top of this layer which influences obviously the interior of the fluid. Consider the Ekman flow of the last section. Firstly the continuity equation (1) implies that since the vertical velocity is zero at the surface then the vertical velocity at the top of the boundary layer is simply the vertical integral of the horizontal convergence $-(u_x + v_y)$ within the boundary layer. Additionally our simplifying assumption that $v_g = 0$ implies that the pressure does not vary in the x direction and so u_g is also independent of x. Using this and integrating the Ekman spiral equations over the boundary layer we obtain to a good approximation that

$$w_{top} = \zeta_g \left(\frac{1}{2\gamma} \right)$$

where the geostrophic vertical component of the vorticity is given by

$$\zeta_g = -\frac{\partial u_g}{\partial y}$$

This equation is a good approximation for practical situations as well as our highly idealized Ekman spiral. In the Southern Hemisphere the sign reverses. In the case of vortices it implies that clockwise circulations in the Northern hemisphere (cyclones) have have an upward directed vertical velocity at boundary layer top while the opposite holds for anticyclones. In the Southern Hemisphere anti-clockwise circulations have this property but are still called cyclones there (more detail in the next lecture). This pumping effect of the boundary layer is an important dynamical effect in many different situations and is unique to rotating flows.