Atmospheric Dynamics

Lecture 2: The forcing for the primitive equations and physical parameterization

1 Summary of primitive equations with usual approximations

The approximations deployed commonly in large scale atmospheric applications are the hydrostatic and the neglect of various Coriolis and spherical coordinate terms detailed in section 6.4 of Lecture 1. The three momentum equations are therefore

$$\begin{aligned} \frac{du}{dt} - 2\Omega v \sin\varphi &= -\frac{1}{\rho r \cos\varphi}\frac{\partial p}{\partial \lambda} + F_x \\ \frac{dv}{dt} + 2\Omega u \sin\varphi &= -\frac{1}{\rho r}\frac{\partial p}{\partial \varphi} + F_y \end{aligned} \tag{1}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{2}$$

The heat equation

$$\frac{dQ}{dt} \equiv F_T = c_p \frac{dT}{dt} - \frac{\alpha T}{\rho}\frac{dp}{dt} \tag{3}$$

The moisture equation

$$\frac{dq}{dt} = F_\gamma \tag{4}$$

The equation of continuity

$$\frac{1}{\rho}\frac{d\rho}{dt} + \nabla \cdot \overrightarrow{u} = 0 \tag{5}$$

and finally the equation of state

$$\rho = \rho(q, T, p) \tag{6}$$

Thus we have seven equations in the seven unknowns $(u,\ v,\ w,\ \rho,\ p,\ T,\ q)$. But as you will note forcing terms have been inserted in several of the equations which represent certain unspecified physical processes which need to be specified in terms of our seven unknowns. We now discuss in detail the various physical processes which cause these forcing terms and how they are usually represented in atmospheric models.

2 Diabatic heating F_T

2.1 Radiation

Solar radiation impinges on the top of the atmosphere at a roughly constant rate (the Solar constant). This radiation is either absorbed by the earth and then reradiated as terrestial radiation or else it is reflected directly back to space (by the surface or by clouds). The solar radiation spectrum peaks in the visible range whereas the terrestial radiation peaks in the infrared and so they are referred to respectively as short and long-wave radiation. Their differing wavelengths reflect their differing thermal origins (one on the solar surface and the other from the earth). Within the atmosphere short-wave radiation is absorbed primarily by ozone (in the stratosphere) and aerosols (dust). It is absorbed at the surface by both land and ocean. Long-wave radiation is absorbed primarily by water vapor but also by carbon dioxide and other trace gases.

Within the atmosphere the presence of long-wave absorbers (mainly moisture) ensures that the surface of the earth is significantly warmer than would otherwise be the case. This is known as the *greenhouse* effect. Consider Figure 1 which represents an idealization of this situation.

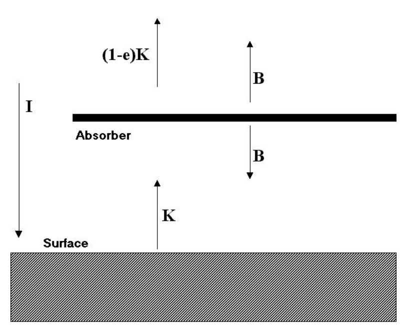

Figure 1: Idealization of the atmospheric greenhouse effect.

where the symbols represent the radiative fluxes and e is the fraction of the upward long-wave radiation absorbed. Unless the flux into an object equals the flux out then that object is not in radiative equilibrium and its temperature will adjust. Let us assume for the sake of argument that our system is in equilibrium (see however below). This means that for the surface we must have

$$K = I + B$$

while for the system as a whole we have

$$I = B + (1 - e)K.$$

Subtracting the equations leads to

$$K = \frac{I}{1 - e/2}$$

Using the black-body formula that the temperature of a body with a certain incident radiative flux is proportional to T^4 we obtain the surface temperature T_s in terms of the temperature T_{na} that would apply in the absence of absorbers

$$T_s = \left(\frac{1}{1 - e/2} \right)^{1/4} T_{na}$$

If $e = 1$ then the greenhouse temperature is approximately 20% above the no atmosphere case which amounts to roughly $50°K$. Note that in this case the absorber temperature is obviously T_{na}. This is a rough model of the radiative equilibrium for the deep tropical surface and atmosphere where the absorber is taken as the troposphere.

In reality the absorbers (mainly moisture) vary strongly as a function of height but the general principle remains the same. The calculation for the real atmosphere is known as a *radiative transfer* computation and involves the precise values of e applicable to the concentration of absorbers as well as the (radiation) frequency dependency. When this is performed the resulting radiative equilibrium temperature profile in the vertical usually roughly resembles the solid line in Figure 2.

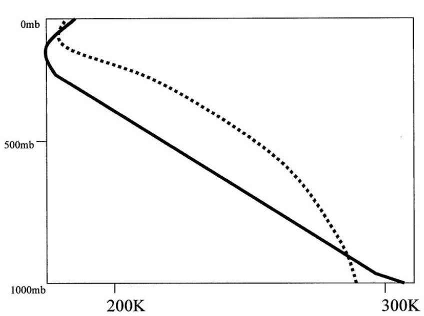

Figure 2: Observed (dotted) and radiative equilibrium (solid) vertical profile of temperature within the atmosphere.

Also drawn on this Figure as a dotted line is the observed temperature profile. Clearly the atmosphere is not in radiative equilibrium and instead, over most of the troposphere, radiation is a *cooling influence*. The reason this happens is because the radiative profile is unstable to convection of two types which we will discuss in more detail below. The radiative transfer calculation is known in principle highly accurately. Unfortunately what is not known very accurately is the physics controlling the nature and position of the atmospheric reflectors and absorbers. The greatest difficulty is usually with predicting the position and composition of clouds.

2.2 Convection

2.2.1 Dry Static Stability

Particular vertical profiles of density in fluids may or may not be gravitationally stable (the atmospheric radiative equilibrium is in general not). An obvious way to test this is to consider two arbitrary parcels of the fluid and move one vertically (and adiabatically) up to the height of the other. If it is less dense than the unmoved parcel then the system is gravitationally unstable and the parcels will exchange vertical ordering given a very small perturbation of the system (and usually very rapidly too). Such an exchange is called *convection* and usually occurs on small horizontal scales. We can derive a criteria for determining if a given profile is unstable or not: The differential form of the heat equation (3) without heating can be combined[1] with the hydrostatic equation (2) to give the temperature equation for an adiabatic ascent of a parcel.

$$\frac{dT}{dz} = -\Gamma \equiv \frac{g\alpha T}{c_p}$$

where Γ is called the dry adiabatic lapse rate. For the atmosphere where $\alpha = T^{-1}$ to a good approximation (since it is close to an ideal gas) we have $\Gamma = g/c_p \approx 10^\circ K$ *per kilometer*. Notice from Figure 2 that the radiative equilibrium profile temperature decreases significantly more rapidly with height than this. If we compare the density of the parcel raised adiabatically with another remaining in place at this higer level then the equation of state tells us that since the two parcels have the same pressure that the major determinant of density difference is the temperature difference between

[1] after conversion to a form to show variations in z

them. In the atmosphere one can ignore to first degree the moisture (tracer) effect on density and the condition for stability is therefore simply that the temperature of the atmosphere not drop with height at a rate greater than the dry adiabatic lapse rate Γ. If this condition is not met then parcels can rise with impunity (i.e. if they are perturbed very slightly) and the vertical column becomes unstable. A process known as convection then results in the column overturning or mixing until stability is restored. In the ocean at very high latitudes the massive cooling of the ocean by the atmosphere results in the surface water becoming colder than that at depth and then the vertical column becomes unstable and oceanic convection takes place. In the atmosphere the convection process mentioned is not the most important form of convection because of the presence of moisture and its associated phase changes which add heat to parcels thereby changing their stability properties.

2.2.2 Moist static stability

In general the atmosphere can only hold a particular amount of water vapor at a given temperature and pressure. This amount is referred to as the saturation specific humidity q_s. The greater the temperature of an air parcel the greater its saturation specific humidity. If the temperature of a parcel drops because of vertical motion so that its moisture content $q > q_s$ then condensation takes place. This involves the conversion of vapor to liquid and the release of latent heat. The amount of heat per unit volume of moist air is given by

$$dQ = L_v \rho dq$$

where L_v is called the latent heat of vaporization and has the large approximate value of $2.5 \times 10^6 Jkg^{-1}$. In the tropics there is about $20g$ of moisture per kilogram of air (which occupies about $1m^2$). If all this was condensed (as happens in a thunderstorm with updrafts) then around $50,000$ joules would be released. Clearly this is a non-trivial amount of heat and it turns out that latent heating is the engine for most tropical circulations such as hurricanes and the like.

Consider now a saturated column of air. The stability properties here are clearly different to the dry case considered previously. If a parcel is lifted now a certain amount of latent heat will be released as the excess moisture is condensed. The parcel will thus be less dense when it is compared with the upper parcel relative to the case of dry adiabatic parcel ascent. It follows that a column may be stable with respect to dry processes but not with respect to moist ones. Such a scenario is called conditional stability and typically holds on average in the troposphere because saturated columns (clouds!) are not always achievable in a given locale. One can define a so-called pseudo-adiabatic lapse rate Γ_s which represents the rate at which temperature would drop in a saturated parcel if it was lifted with only heat being added through condensation. Typically this has values of around $5-6^{\circ}K$ *per kilometer*. i.e. considerably less than the dry value.

One feature of moist convection makes it very different to the dry process and that is its "directionality". A saturated parcel can be raised and become more buoyant than its surroundings due to latent heat release but the opposite does not occur as it does in the dry case. In general if one lowers a parcel it becomes unsaturated and there is no absorption of latent heat. The reverse process can occur under very particular circumstances if it is able to retain liquid water and re-evaporate it as it descends. Such a process can occur in a thunder cloud and is called a downburst.

2.2.3 Dynamical implications of moist convection

The tropical atmosphere is strongly influenced by the release of latent heat in the massive rainfall events that take place there. In general, a heat source in a fluid subject to gravitation, drives an overturning cell with uplift occurring in the vicinity of the heating. Due to continuity this implies convergence in the lower atmosphere and divergence aloft. In a later lecture we shall derive a mathematical model of this process.

There are two major tropical cells caused primarily by this heating. One occurs in the meridional (latitude) direction and is called the Hadley Cell. The other occurs in the zonal (longitude) direction and is called the Walker Cell. To see why this occurs consider Figure 3 which maps the major areas of moist convection in the tropics for last month (a typical situation).

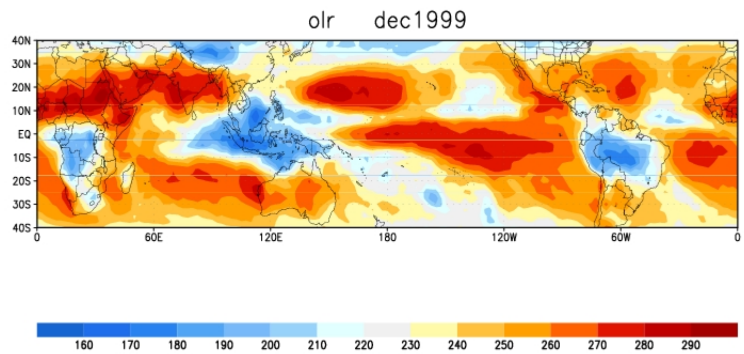

Figure 3: A typical map of global outgoing long wave radiation (OLR).

The Figure displays the temperature of objects emitting long-wave radiation to space (Outgoing Long-wave Radiation or OLR). Low temperatures are associated with cloud tops while high temperatures are associated with the surface. Thus the lower the temperature in general the greater the tropical rainfall activity and hence the greater the latent heating. Notice that most heating occurs near the equator but also in very specific locations (particularly in the Indonesian region). Overturning cells therefore develop between the equator and the subtropics (notice the absence of rain here due to the induced downward motion) but also between equatorial regions of differing longitudes. Note the absence of rain in the eastern equatorial Pacific again caused by downward motion.

Another significant feature of moist convection is that, regarded as a heat engine, it is able to automatically supply its own fuel: The convergent circulation induced in the lower atmosphere means that the water vapor that is lost during rainfall can be replaced by vapor imported horizontally from large distances away. The distribution of water vapor in the vertical shows a strong exponential fall off from the surface (see below) so therefore the lower level moisture convergence is much more effective at importation than the moisture export due to upper level divergence. This positive feedback process is responsible for the persistence and occasional rapid growth of tropical rainfall systems with the most spectacular example being the hurricane. It also ensures the ubiquity and dynamical dominance of tropical convection on the Earth.

One of the greatest uncertainties in climate models is the description of moist convection. Clearly it involves a huge range of length and time scales from condensate motion through cloud formation and up to global circulations. Clearly this entire turbulent like process is not resolvable and the effects of the small scale features such as clouds may only be treated in a statistical sense on the larger scales able to be modeled in a climate context. The process by which this is done is called *convection parameterization* and is an area of considerable research and unfortunately (but necessarily) ad hoc formulation. This uncertainty also affects the radiation calculation since moisture in all its forms is radiatively active.

3 Turbulent Transfer

3.1 Vertical

The surface of the earth, whether it be the ocean or the land, represents a sharp discontinuity in fluid properties. This means that it is an effective generator of turbulent motion in both atmosphere and ocean. The turbulence is generated by shears in currents and winds and by convection caused by the often quite different ocean/land and atmosphere temperatures. In the atmosphere this turbulence is typically greatest in a region referred to as the atmospheric boundary layer which is about $1km$ thick. In the ocean the equivalent region is known as the mixed layer and is typically $30 - 100m$ in depth. In these zones there are typically very strong vertical fluxes of momentum, heat and tracers. At the earth's surface these fluxes match and this exchange between ocean and atmosphere is fundamental to understanding the climate system. Unfortunately, as with moist convection, it is not practical to resolve the large range of scales associated with this motion in climate models. Resort is made to (turbulence) *parameterization* again and this also remains an area of significant uncertainty in models.

The forcing terms for the primitive equations due to turbulence are typically written in terms of the fluxes of momentum, heat and tracers. Thus the flux of momentum is the time rate of change of momentum passing upward through a unit horizontal area. This is commonly called the stress $\overrightarrow{X}$ and has units of Newtons per square meter. The accumulation of momentum in a unit volume is easily seen to be the vertical gradient of the stress so the forcing term may be written

$$\overrightarrow{F} = \frac{1}{\rho}\frac{\partial \overrightarrow{X}}{\partial z}$$

where we are now using z as our vertical coordinate. In a similar manner the heat flux H is specified in units of Watts per square meter and then the forcing term for the heat equation may be written

$$F_T = \frac{1}{\rho}\frac{\partial H}{\partial z}$$

A similar equation applies for tracers. The key to parameterizing the vertical turbulence obviously lies in the specification of the fluxes. A typical formulation is to assume that the flux is proportional to the vertical gradient of the quantity of interest e.g. for momentum

$$\overrightarrow{X} = \rho\nu\frac{\partial \overrightarrow{u}}{\partial z}$$

and then specify the coefficient ν in terms of the local susceptibility of the fluid to turbulence or mixing. This can sometimes be measured as a function of the so-called gradient Richardson number (a dimensionless number)

$$Ri = \frac{\rho}{g}\frac{|\partial \overrightarrow{u}/\partial z|^2}{\partial \rho/\partial z}$$

Note that if ν is treated as a constant we get a Laplacian term for the turbulent transfer. This is known as a laminar Ekman flow.

The fluxes at the fluid interface are critical for the large scale dynamical coupling between ocean and atmosphere and many attempts have been made to measure them. Empirical formulae exist and a typical set are

$$\begin{aligned} \overrightarrow{X} &= \rho c_u \, |\overrightarrow{u}| \, \overrightarrow{u} \\ Q_s &= \rho c_H \, |\overrightarrow{u}| \, (T_s - T) \\ E &= \rho c_E \, |\overrightarrow{u}| \, (q_s - q) \end{aligned} \qquad (7)$$

where the c coefficients are dimensionless numbers that are close to unity. The variables $\overrightarrow{u}$, T and q are the velocity, temperature and humidity at a typical measurement height above the surface (around $10m$). The s subscripted variables are the surface values while E measures the evaporation rate and Q_s the sensible (turbulent) heat flux. Note that evaporation is an upward flux of moisture in the atmosphere but of heat in the ocean. In the tropics the fluxes $\overrightarrow{X}$ and E are very important forcing terms for determining the ocean circulation as we shall discover in later lectures.

3.2 Horizontal

In both atmosphere and ocean horizontal eddying is an important process however the scales are very different. The typical atmospheric eddies of importance are the synoptic storm systems and these are usually quite well resolved in climate models as they have a typical dimension of $1000km$. Of course convective systems are more problematic because of the convection parametrization issue. In the ocean the typical horizontal eddy size is much smaller (around $20 - 50km$) and therefore they are often crudely parameterized in an analogous fashion to the vertical turbulence. Recent large increases in computing power has meant that ocean resolution is becoming more practical and many so-called *eddy resolving* ocean models are being explored.