Assignment 2: Atmospheric Dynamics

All questions require detailed explanations of answers.

1 Vertical Modes

Consider the linearization about a state of rest for the compressible atmosphere using pressure as a vertical coordinate (ignore moisture effects as in Lecture 5). Assume for simplicity that the static stability satisfies the equation

$$\frac{S(p)}{p} \equiv K \sim 10^{-8} \tag{1}$$

which is accurate for the mid troposphere (but less so elsewhere). Consider the separation of variables problem of the Lectures and derive a simple second order ODE for the vertical structure function for vertical velocity. Using the boundary conditions for these equations discussed in Lectures, derive conditions for eigenvectors and eigenvalues for this Sturm Liouville problem. Using the numerical value from equation (1) deduce an approximate solution to this eigenvalue/eigenvector problem. Sketch the structure functions for the first 3 modes (vertical velocity and geopotential/horizontal wind) and work out the approximate value of the corresponding shallow water speeds (assume that the bottom of the atmosphere has pressure 1 $bar = 10^5 Nm^{-2}$ with density 1.2 $(kg)m^{-3}$).

2 Horizontal Waves

Suppose there exists a forcing at fixed frequency for the linearized equations about a state of rest. Deduce limits on latitude for which this forcing can induce propagating Rossby and Poincare waves.

3 Rossby Waves

Derive an expression for the group velocity for a Rossby wave of maximum frequency with respect to zonal wavenumber. Sketch (use matlab) the time evolution of this wave in the (x, y) plane with choices of f_0 corresponding to $45°N$ and a shallow water speed of $60ms^{-1}$.