Study aid for the final exam

Instructions and information about the final

- The exam is take-home. It will be posted at 10am Monday, December 21. Answers must be uploaded by 10am Tuesday December 22. The exam should take about two hours to complete.
- You may consult notes or books. You may not consult other online sources or people. You may not communicate about the exam with anyone until you have uploaded your answers.
- You will receive 20% credit for a blank answer. Points may be deducted from this if a given answer is wrong. If you give two answers, you will get points for the correct answer and lose points for the wrong answer. Cross off or delete (if typing) anything you think is wrong.
- Explain your answers as fully as time permits.
- Email the instructor immediately if you have a question about the exam.
- The actual exam will be shorter than this.

True/False

In each case, say whether the statement is true or false. Explain your answer in a few words or sentences as appropriate. You may lose points even if the answer is correct if the explanation is missing or incorrect.

1. If a random walk process must have Gaussian steps in order to converge to a Brownian motion process.

Multiple Choice

In each case, select the one answer you think is most correct. Explain your answer in a few words or sentences as appropriate. You may lose points even if the answer is correct if the explanation is missing or incorrect.

1. Which of the following is not a martingale? W_t is mean zero Brownian motion.

(a) $X_t = W_t^2 - t$

(b) $X_t = W_t^4 - 3W_t^2$

(c) $X_t = tW_t$.

(d) $X_t = \int_0^t e^s dW_s$

Full answer questions

1. Suppose W_t is a standard Brownian motion and $X_t = a_t W_{t^2}$. Find a_t so that X_t is a standard Brownian motion.

2. Suppose (X, Y) is jointly Gaussian with mean zero and covariance $\begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$.

   Find β so that $X - \beta Y$ is independent of Y.

3. Suppose U_n are independent random variables with $U_n = 0$ or $U_n = \pm 1$ and $\Pr(U)n = 0) = \frac{1}{2}$ and $\Pr(U_n = -1) = \Pr|(U_n = 1) = \frac{1}{4}$. Define

$$S_n = \sum_{k=1}^{n} U_k \ .$$

   Suppose R is a large number. Estimate the distribution of the first hitting time

$$N = \min \{ n \mid S_n = R \} \ .$$

   Use a Brownian motion scaling to express this, approximately, in terms of a hitting time for Brownian motion.