# Assignment 1.

Given January 22, due January 29.

**Objectives:** (1) To warm up to programming with good habits, (2) To illustrate basic numerical ideas.

We want to study the function

$$f(x) = \int_0^1 \cos\left(xt^2\right) dt \quad . \tag{1}$$

**Step 1:** Write a program to estimate $f(x)$ using a panel integration method. The program should be modular, well documented, robust, and clean. At the lowest level should be a procedure that takes as input $x$ and $n = 1/\Delta t$ and returns the approximate integral with that $x$ and $\Delta t$ value. This routine should be written so that another person could easily substitute a different panes method by changing a few lines of code.

**Step 2:** Verify the correctness of the program by checking that it gives the right answer for small $x$. We can estimate $f(x)$ for small $x$ using a few terms of its Taylor series. This series can be computed by integrating the Taylor series for $\cos(xt^2)$ term by term.

**Step 3:** With $x = 1$, do a convergence study to verify the second order accuracy of the trapezoid rule and the fourth order accuracy of Simpson's rule.

**Step 4:** Write a routine that uses the routine from Step 1 and Richardson estimation to find an $n$ that gives $f(x)$ to within a specified error tolerance. The input should be $x$ and the desired error bound. The output should be the estimated value of $f$ and the number of points used. This routine should be robust enough to quit and report failure if it is unable to achieve the requested accuracy.

**Step 5:** For large $x$ we have the approximation

$$f(x) \sim \sqrt{\frac{\pi}{x}} + \frac{1}{2x}\sin(x) - \frac{1}{4x^2}\cos(y) + \cdots \quad . \tag{2}$$

Make a few plots showing $f$ and its approximations using one, two and all three terms on the right side of (2) for $x$ in the range $1 \leq x \leq 1000$. In all cases we want to evaluate $f$ so accurately that the error in our $f$ value is much less than the error of the approximation (2). Note that even for a fixed level of accuracy, more points are needed for large $n$. Why?