Syllabus

Course description

This course discusses numerical methods for solving ordinary and partial differential equations. We follow the topic order in the LeVeque book

We start with static problems involving elliptic PDE, then moving to iterative strategies for solving the resulting systems of equations. The discretization part includes order of accuracy, convergence analysis, stability of discretizations, and techniques for code validation. Accuracy of discrete approximations is proved using discrete analogues of *estimates* (inequalities) that apply to the corresponding PDE. Iterative strategies starts with "pointwise" iterations such as Richardson, Jacobi, and Gauss Seidel. It moves to acceleration methods (conjugate gradients, GMRES, over-relaxation, momentum) and PDE specific ideas (pre-conditioners, multigrid). We discuss finding and using variational principles when available.

Next comes simulation methods for ordinary differential equations, including Runge Kutta and linear multi-step methods. We cover the characteristic polynomial approach to stability theory for linear multi-step methods. We explore advantages and drawbacks of higher order and implicit methods. Stability regions are defined, and worked out for certain schemes.

Finally, we combine this material in a discussion of finite difference methods for time dependent solutions of PDE that model diffusion and wave propagation. New issues include relations between the time and space discretization (CFL conditions) and the nature of the equation systems that arise in implicit methods. The discrete Fourier transform and FFT algorithm are used both for theory and as computational tools.

Students are strongly encouraged to code in Julia, but may choose to use Matlab or C++ or Fortran. It may be frustrating to work with Python because many of the algorithms involve loops that run slowly in purely interpreted languages like Python. There will be sample code posted, which will be in Julia.

Prerequisites

- Multi-variate calculus
- Basic numerical analysis: finite differences, interpolation, order of accuracy, conditioning, floating point arithmetic.
- Numerical linear algebra, such as from *Numerical Methods I*, including factorization, eigenvalues/eigenvectors, condition number, etc.
- Coding experience
- A background in differential equations is very helpful but not required

Reference books

- (**required**) *Finite Difference Methods for Ordinary and Partial Differential Equations*, Randall LeVeque
- (recommended, iterative methods and condition number) *Applied Numerical Linear Algebra*, James Demmel
- (recommended, finite differences, ODE methods) *Numerical Methods*, Germund Dahlquist, Åke Björk
- (recommended, deeper theory for numerical PDE) *Difference Methods for Initial-Value Problems*, Robert Richtmyer, K. W. Morton

The last two are older than other books on this material, but they contain material that is hard to find in more recent books.

Assignments and grading

There will be 7 assignments that will require, coding, design and analysis of methods, and some more theoretical exercises. The last assignment will substitute for a final exam.

Collaboration and academic integrity

Students are allowed to collaborate and use outside resources for the first six assignments. Such exploration and collaboration shall not include code sharing (from internet sources, personal connections, or fellow students). Each student much write up their solutions individually. Students are encouraged to report helpful collaborations and outside materials, both for the benefit of the instructor and to avoid misunderstandings. Collaboration or materials outside the texts and class notes are not allowed for the final assignment.

Allowed	**Not allowed**
Sharing ideas	submitting answers from another student
Using outside and online resources	using answers found online
Help with exercises and debugging	code sharing

Any violation of the academic integrity policies is *cheating.* A first offense may just draw a warning, if it involves a judgement call, but even a first offense may lead to getting a zero for an assignment. Subsequent offenses may lead to grade penalties.

	topics	sections
1	discrete Laplace equation, accuracy, variational principle, Gauss Seidel	1.2 to 1.5
2	discrete Fourier transform, FFT, fast solution, condition number of the discrete Laplace operator, implications for gradient descent	1.2 to 1.5
3	variable coefficient and advection problems, orthogonalization methods: conjugate gradients and GMRES	1.2 to 1.5
4	PDE specific strategies: pre-conditioners, multigrid	1.2 to 1.5
5	higher order and spectral discretization	1.2 to 1.5
6	ODE initial value problem, Euler and Runge Kutta methods	1.2 to 1.5
7	linear multistep methods (Adams Bashforth, Nyström)	1.2 to 1.5
8	stability, stability regions, convergence proofs	1.2 to 1.5
9	implicit methods, predictor/corrector schemes, solution/iteration strategies	1.2 to 1.5
10	initial value problem for parabolic PDE, method of lines discretization, time step constraints	1.2 to 1.5
11	initial value problem for hyperbolic PDE, method of lines discretization, time step constraints	1.2 to 1.5
12	implicit and IMEX schemes, exponential integrators	1.2 to 1.5
13	stability and discrete norms, Lax convergence theorems	1.2 to 1.5