

Introduction

1 Introduction

This is a set of 13 classes on numerical solution of differential equations given in the Spring semester of 2024. It is intended for two overlapping groups: people who want to solve differential equations to learn about the solutions (applications), and people who want to create new methods or figure out why existing methods do what they do (theory). These goals go together. Many differential equation applications are so hard that off-the-shelf solvers will not work well without at least some customization. A user needs to know about the problem theory and the methods to customize effectively. Going the other way, new methods and new theory should be motivated by specific issues in specific application experience.

The level is first year PhD students in math or computer science. The most important prerequisite is our class Numerical Methods I, which covers basic numerical analysis (order of accuracy, interpolation, differentiation, conditioning, and numerical linear algebra (SVD, symmetric and non-symmetric eigenvalue problems, LU, QR, etc.)). General mathematical prerequisites include good multi-variate calculus, beyond three dimensional vector calculus and what is called *mathematical maturity*, which is the ability to read and apply mathematical reasoning at the necessary level.

Class assignments will involve some paper-and-pencil exercises (prove this, derive a formula for a method that does that, etc.) and some programming exercises and numerical experiments. Some of the assignments will require too much computing power to be done in an interpreted language such Python. Best would be to use a compiled language such as C/C++ or Fortran. Julia seems to be significantly slower, but still fast enough for the assignments in this class. I don't know about Matlab. If you use C/C++ or Fortran, you will need to use Python, Matlab or Julia (poor third choice for this purpose) for post-processing and visualization.