Assignment 5.

Given December 8, due end of finals week.

Objective: To work with advanced Monte Carlo methods

The portfolio is the same as that of Assignment 4. Let V be the value of the portfolio at six months, as before. We want the following quantities:

$$\begin{aligned} \overline{v} &= \mathrm{E}\,(V) && (1) \\ \sigma_V^2 &= \mathrm{E}\left((V-\overline{v})^2\right) && (2) \\ v_p & \quad\text{so that } \Pr(V < v_p) = p \ . && (3) \end{aligned}$$

We can estimate v_p using "order statistics"[1]. Suppose $V_1, \ldots, V_N$ are the N sample values of V resulting from N independent paths. The notation $V_{(k)}$ refers to the same N numbers, but in increasing order: $V_{(1)} \leq V_{(2)} \leq \cdots$. Now choose k so that $k/N \approx p$, and use as estimator

$$\hat{v}_p = V_{(Np)} \ . \quad (4)$$

If $f(v)$ is the probability density function for V, the variance of $\hat{v}_p$ is approximately

$$\frac{p(1-p)}{Nf(v_p)} \ .$$

Try to verify this computationally.

Use the SDE solver from Assignment 4 to estimate $\overline{v}$, σ_v^2, and v_p with $p = 1\%$. Then redo this using as control variate the same portfolio but with European style options, for which the expected value can be found analytically. Warning: this is not the Black-Scholes "rational" price. This does not apply to v_p. Finally, extimate the perturbations of these three quantities when
(i) the growht rate of stock 1 goes to .35.
or
(ii) the covariance of $X_1(1)$ and $X_2(1)$ goes from .84 to .9.

Use the "different paths" method, the "same paths" method, and the "score function" method. Comment on the results. Warning: this is a class in computing, so you should comment on computational advantages and disadvantages, not on the financial significance of the answers.

When you have finished all this, try to determine a suitable way to *relax and recover from a tough semester! You're done.*

[1] In statistics, the term "order statistic" refers to a statistic that depends on the samples but not on the order in which whey are obtained.