Assignment 4.

Given October 22, due November 5.

Objective: To experiment with Monte Carlo and error bars.

Step 1: Generate iid standard normal random variables, X_j, $j = 1, 2, \cdots$, using the Box Muller method. Compute the "bin counts":

$$N(k,n) = \#\left\{X_j \in (x_k - \Delta x/2, x_k + \Delta x/2) \ , \ 1 \leq j \leq n\right\} \ .$$

The interval $(x_k - \Delta x/2, x_k + \Delta x/2)$ is bin k, and $N(k,n)$ is the number of random numbers from the first n that fall in bin k. The expected number of numbers in bin k is

$$e(k,n) = E\left[N(k,n)\right] \approx n\Delta x \frac{1}{\sqrt{2\pi}} e^{-x_k^2/2} \ .$$

Choose Δx small and then n large enough so that the bin counts are pretty large for small k. Compare the empirical bin counts $N(k,n)$ with the expected values. Make a plot.

Step 2: Estimate

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x^6 e^{-x^2/2} dx = E\left[X^6\right] = 15$$

using Monte-Carlo and iid standard normals from Box Muller. Choose a sample size so that the estimate is within 1% of 15 with 95% confidence. That is, increase the number of samples until the confidence interval about the estimated value allows errors less than 1% of the estimated value. Note: a statistician would correctly point out that this procedure may not guarantee 1% of the answer with 95% confidence.

Step 3: An insurance company is insuring against five natural disasters. The probability of distaster D_i is $p_i = \Pr(D_i)$. The disasters are independent events. If D_i happens, the company must pay S_i, which is a Gaussian random variable with mean m_i and standard deviation σ_i.

(a) Make a histogram of the probabilities of various payout amounts.

(b) Compute the probability of a payout of $\$10^9$ or more to within 5%.

i	1	2	3	4	5
m_i ($\times\$10^6$)	251	194	620	432	57
σ_i ($\times\$10^6$)	68	95	119	120	10
p_i	.1	.3	.2	.06	.8