Assignment 2.

Given October 9, due October 23.

Objective: To explore the forward Euler method as a valuation tool.

The Bergman Goodman (BG) pricing model for a European put on a non dividend paying bank stock is a two factor model that takes into account stochastic fluctuations in the stock price and the "risk free" short term interest rate. The interest rate, $r(t)$, is governed by the Vasicek model

$$dr = a(r_0 - r)dt + r\sigma_r dZ_r \quad , \tag{1}$$

where r_0 is the natural interest rate, a is a time constant governing the rate of return of the interest rate to its natural value, $Z_r(t)$ is a Wiener process that drives fluctuations in the interest rate, and σ_r is the volatility of the interest rate. The stock price, $S(t)$, is governed by

$$dS = rSdt + S\sigma_S dZ_S + \frac{mS}{r} dr \quad , \tag{2}$$

where $Z_S(t)$ is a Wiener process independent of Z_r. The only new thing is the last term, which quantifies our assumption that any change in interest rates has an immediate effect on the stock price. From the theory of stochastic processes, we know that the expected discounted value of the put, $f(S, r, t)$, satisfies the BG equation:

$$\begin{aligned} \partial_t f \quad &+ \quad \left(rS + \frac{amS(r_0 - r)}{r} \right) \partial_S f + a(r_0 - r)\partial_r f \\ &+ \quad \frac{1}{2} \left(S^2 \left(\sigma_S^2 + m^2\sigma_r^2 \right) \partial_S^2 f + 2mrS\sigma_r^2 \partial_r \partial_S f + r^2 \sigma_r^2 \partial_r^2 f \right) = rf \quad . \end{aligned} \tag{3}$$

The expiration and possible exercise time of the put is T. At this time, the value of the put is known to be

$$f(S, r, T) = \max(K - S, 0) \quad ,$$

where K is the strike price. The problem parameters for this particular stock are (with time measured in years)

$$r_0 = .05 \quad , \quad a = 1 \quad , \quad \sigma_S = .5 \quad , \quad \sigma_r = .4 \quad , \quad m = -2 \quad , \quad T = .5 \quad , \quad K = 20 \quad .$$

Use the forward Euler method with constant $\Delta S/S$ and $\Delta r/r$ on a computational domain $S_{\text{min}} \leq S \leq S_{\text{max}}$, $r_{\text{min}} \leq r \leq r_{\text{max}}$ with suitable boundary values on the boundary. Do a convergence study to determine the order of accuracy of your method. Choose a time and space step so that the error in $f(S = 20, r = .04, t = 0)$ is less than 1%. Prepare a few well chosen graphs to illustrate the answer and the performance of the method. Describe how you tested your code to make sure it given the right answer.