Calculus I, Sections 4, 5, 6.

Practice for Quiz 1

If you know your stuff, this should take about an hour. The actual quiz will be about a quarter this long and should take about fifteen minutes.

1. True or false: in each case, state whether the statement is true or false and give a brief explanation. For a statement to be true, it must be true in every case. An easy way to show a statement is false is to give a *counterexample*, a single case in which it does not hold.

a. If $f(x)$ is a continuous at $x = 2$, then f is differentiable at $x = 2$.

b. If $\lim_{x\to a} u(x)$ and $\lim_{x\to a} v(x)$ exist, then $\lim_{x\to a} u(x)v(x)$ also exists.

c. The function $f(x) = \sqrt{|x|}$ is differentiable at $x = 0$.

2. Evaluate the following limits:

a. $\lim_{t\to 3} t^2 + \dfrac{1}{t}$.

b. $\lim_{x\to 1} \dfrac{2x^2 + x + 2}{3x - 4}$.

c. $\lim_{x\to -2} \dfrac{x^2 + 3x - 2}{x^2 - 4}$.

d. $\lim_{t\to 0} \dfrac{1}{1 + \frac{1}{1+\frac{1}{x}}}$. Hint: This is hard. Try multiplying top and bottom by x a few times.

e. $\lim_{h\to 0} \dfrac{f(x+3h) - f(x)}{h}$, where $f'(x) = 2$.

3. Compute the derivative of $f(x) = (x^2 + 1)(x^2 - 1)$ in two ways and check that the result is the same:

a. Multiply out the expression $(x^2+x)(x^2-x)$ and apply a differentiation formula to each term.

b. Find the derivative of $u(x) = x^2 + x$ and of $v(x) = x^2 - x$, then use the product rule.

4. Suppose $n(t)$ is the number of bacteria (measured in millions) in a container at time t, and $u(t)$ is the rate at which food is added to the container (measured in grams per hour) and consumed by the bacteria. Let R be the rate at which $n(t)$ is increasing and S be the rate at which u is increasing. The amount rate at which a single bacterium consumes food is $f(t) = u(t)/n(t)$.

a. Express R and S as derivatives.

b. Find an expression for the rate of change of $f(t)$ in terms of n, u, R, and S.

5. Use the formal rules of differentiation to find the derivatives of each of the functions below. The derivative should be correct at every place where the function is differentiable.

a. $f(x) = x^3 - 5x^2$.

b. $f(x) = \dfrac{x+2}{x-3}$.

c. $f(x) = \left(\dfrac{1}{x^4} - x^4\right)^2$ (hint: write this as a product and use the product rule).

6. Make a careful sketch of the graph of the function $f(x)$ with values

x	0	.2	.4	.6	.8
f	.20	.36	.60	.92	1.32

a. Using a ruler or straight edge, draw the line tangent to the curve at $x = .4$. Read the slope of the line from the graph.

b. Estimate the slope using an appropriate $\Delta f/\Delta x$.

c. Give one or more reasons for the answers to a. and b. to be a little different.