Fall 2004 Calculus I, sections 4, 5, 6, Courant Institute of Mathematical Sciences, NYU.

Homework 9, due November 15

Self check (not to hand in, answers are in the back of the book):

Section 5.7: 1, 5, 7, 17, 21, 27, 39, 49.

In all integrals, check the result by differentiation. The answer is incomplete without this.

To hand in:

Section 5.7: 2, 4, 8, 16, 26, 28, 42, 50.

In all integrals, check the result by differentiation. The answer is incomplete without this.

More problems (to hand in)

1. Use the formulas $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$, and $\sin^2(x) + \cos^2(x) = 1$, and $a = b = t$, to find a formula for $\sin^2(t)$ in terms of $\cos(2t)$. This is called a *double angle* formula. It is useful in integration.

2. Explain how to draw the graph of $f(t) = a + b\cos(ct)$ using the horizontal lines at a, $a - b$, and $a + b$. How do you take c into account? Use this method to make a graph of $\sin^2(t)$ using the double angle formula from part 1.

3. Explain how to make a rough sketch of the graph of a function $f(t)^2$ on top of a graph of $f(t)$. Note that $f(t)^2$ has a local minimum wherever $f(t)$ crosses the x axis. Make a graph of the functions $f(t) = t^2 - 1$ and $f(t)^2 = \left(t^2 - 1\right)^2$ on top of each other. Explain the features (the local minima) of the graph of $f(t)^2$ in this way.

4. Apply the method of part 3 to make graphs of $\sin(t)$ and $\sin^2(t)$ in the same picture. Explain the local minima, the range of variation, and the period of $\sin^2(t)$ from this picture.

5. Compare the graphs of $\sin^2(t)$ from parts 2 and 4.

6. The average of a function over the interval (a, b) is $\frac{1}{b-a}\int_a^b f(t)dt$. Show that this formula agrees with intuition when f is a constant function and when f is a linear function (when the average is the value at the mid point).

7. The *long term average* is the limit (if there is one) of the average as $a \to -\infty$ and/or $b \to \infty$. Find the long term average of $f(t) = \sin^2(t)$ both by working the integral and finding the limit and by examining the graphs from part 2 and part 5.