

Fall 2004 Calculus I, sections 4, 5, 6, Courant Institute of Mathematical Sciences, NYU.

Homework 3, due September 27

Self check (not to hand in, answers are in the back of the book):

Section 3.4: 1, 3, 17, 19, 38, 43.

Section 3.5: 1, 5, 13, 17, 21, 23, 25, 29, 41, 43, 71.

To hand in:

Section 3.4: 2, 6, 16, 18, 20, 24, 44, 48.

Section 3.5: 2, 10, 16, 20, 22, 26, 30, 42, 46, 72.

More problems (to hand in)

1. A balloon expands in hot weather. The balloon manufacturer measured the radius of the balloon as a function of temperature.

| | | | | | |
|---------------|------|------|------|----|------|
| Temp (deg. C) | 0 | 10 | 20 | 30 | 40 |
| Rad. (cm) | 37.8 | 38.3 | 38.7 | 39 | 39.2 |

The measured temperature on a certain day was

| | | | | | | |
|---------------|-------|------|------|------|------|-----|
| Time | 12 pm | 1 pm | 2 pm | 3 pm | 4 pm | 5pm |
| Temp (deg. C) | 21 | 29 | 34 | 35 | 30 | 22 |

About how fast (in cm. per hour) was the balloon expanding at 2pm?

2. The frequency of vibrations of a vibrating violin string is given by

$$f = \frac{1}{2L} \sqrt{\frac{T}{\rho}}$$

where L is the length of the string, T is its tension, and ρ is its linear density. Find the rate of change of the frequency with respect to

- a. the length L (when T and ρ are constant),
- b. the tension T (when L and ρ are constant),
- c. the linear density ρ (when L and T are constant).
- d. Your answer from part **a.** is called the partial derivative of f with respect to L and is written $\frac{\partial f}{\partial L}$. Show that

$$\frac{\partial f}{\partial L} = -\frac{f}{L}.$$

The mathematical definitions are:

$$\frac{\partial f}{\partial L} = \lim_{\Delta L \rightarrow 0} \frac{\Delta f}{\Delta L} \quad , \quad \frac{\partial f}{\partial T} = \lim_{\Delta T \rightarrow 0} \frac{\Delta f}{\Delta T} \quad , \text{etc.}$$

where, respectively,

$$\Delta f = f(L+\Delta L, T, \rho) - f(L, T, \rho) \quad , \quad \Delta f = f(L, T+\Delta T, \rho) - f(L, T, \rho) \quad , \text{etc.}$$

Although we write ∂ instead of d to denote the derivative, the definition is the same as the one we have used up to now.