Calculus I, Sections 4, 5, 6.

Practice for the Midterm

1. Compute the following limits. Explain your reasoning. The Midterm will have three questions on limits, some similar to these.

   (a) $\lim_{x\to 0} \dfrac{\sin(x)}{x\cos(x)}$.

   (b) $\lim_{x\to\infty} \dfrac{x}{x+\cos(x^2)}$.

   (c) $\lim_{x\to 0} \dfrac{f(x^2)}{x^2}$, where $f(0) = 0$ and $f'(0) = 3$. Hint: for small x^2, estimate $f(x^2) = f(x^2) - f(0)$ using the derivative approximation and the fact that $f(0) = 0$.

   (d) $\lim_{x\to -1} \dfrac{x+1}{x^3 - 2x^2 - x + 2}$.

2. Evaluate the following. The midterm will have four straight derivative evaluations, some similar to these.

   (a) $f'(x)$, where $f(x) = \dfrac{1+\sqrt{x}}{x}$.

   (b) $f'(\theta)$, where $f(\theta) = \tan(3\theta + \sin(\theta))$. (The answer is not pretty. The midterm problems will involve a bit less algebra.)

   (c) $\dfrac{df}{dt}$ and $\dfrac{d^2f}{dt^2}$ when $t = 0$, where $f(t) = \sin(u(t))$ and $u(0) = 0$, $u'(0) = 1$, and $u''(0) = 2$.

   (d) $f'(x)$, where $f(x) = \dfrac{1}{x^{2n} + x^{-n}}$.

   The midterm will have one harder differentiation question. Here are two samples.

3. A curve in the plane is defined by the equation $x^2 - 2(x+y) + y^3 = 3$. Find the only critical point on the curve (a point where $\frac{dy}{dx} = 0$). Determine whether this is a local maximum or a local minimum. (Hint: use implicit differentiation to evaluate the first and second derivatives.).

4. A function $x(t)$ satisfies the equation $x(t)^2 + \left(\dfrac{dx}{dt}\right)^2 = E$, where E is a constant. Find a formula for $\frac{d^2x}{dt^2}$ in terms of $x(t)$ and $\frac{dx}{dt}$.

The midterm will have 5 word problems of various types. Here are some samples.

5. The ideal gas law relates the pressure, temperature, and volume of an ideal gas and is given by the formula

$$PV = nRT,$$

where P is the pressure, V is the volume, T is the temperature (measured in Kelvin: $K \approx {}^\circ C + 273$). R represents the universal gas constant ($R \approx 8.3$ Joules/(mol K)) and n the number of moles of gas present (assume n is a given constant in the problem). Let us suppose we have a balloon whose volume changes with changes in the temperature according to the table below. Assume the air in the balloon is an ideal gas. How is the pressure changing with the temperature when it is 296 K?

Volume (cm)^3	800.4	801.6	802.5	803.6	804.4
Temperature (K)	294	295	296	297	298

6. A point is moving in a circular path at a constant speed of 4 revolutions per minute. The circle has radius 2 and center $(1, 0)$. Let $R(t)$ be the distance at time t of the point from the origin. Let $v(t)$ be the rate of change of $R(t)$.

   (a) Write a formula for the coordinates of the point at time t using trigonometric functions sin and cos. Note that there can be slightly different answers depending on where you place the point at time $t = 0$ and whether you have it move clockwise or counterclockwise.

   (b) Write a formula for $R(t)$ and simplify (if you don't simplify, the next part will take much longer).

   (c) How fast is the distance from the point to the origin changing when it's y coordinate is as large as possible?

7. A cube of water 1.5 inches on each side freezes, increasing its volume by 9%. Approximate the change in the length of each side and compare this to the exact change in the length. How close was your approximation?

8. You have a rectangular piece of cardboard of dimensions 3 ft by 6 ft. By cutting out four congruent squares from the corners, you can fold the piece of cardboard into a box. What size squares should you cut out to make the box of maximum volume?

9. Draw a careful graph of the function $f(x) = \dfrac{1 - x^2}{x^3}$. Include all important features in the graph which may include: intercepts, roots, asymptotes, critical values, and inflection points.