Practice for the Midterm exam, Thursday, March 24.

Midterm exam Instructions and information

- The midterm exam will take the whole class period. Make sure to arrive on time.
- The midterm exam will not be given online.
- The midterm exam will be closed book, closed notes, etc. You may not use any resources during the quiz, except ...
- You may bring and use a "cheat sheet", which is one US standard size ($8\frac{1}{2}'' \times 11''$) piece of paper, front and back. Please upload the cheat sheet with your quiz answers, if you use one.
- Please write as clearly and neatly as possible in a quiz situation. If you scan or photograph a handwritten paper (the most common mode), please do that as well as possible in the quiz setting.
- You will be graded on clarity as well as mathematical correctness. You don't have to use full sentences in each case, but what you write should be grammatical and use mathematical terms and notation correctly. You may use scratch paper that you don't hand in to organize your thoughts. Reasoning is as important as the answer in a theory class like abstract algebra.
- You will get 25% credit for any question or question part that you leave blank. You may lose points for a wrong answer, even if you also give a correct answer. Cross out anything you think is wrong.

Questions

This variety of questions should help you study. The questions focus on material after the quiz, but the midterm exam will have questions from the first part of the course. You should also review the assignment questions.

1. Show that the $\mathbb{Z}$ has no automorphisms except the identity. Identify the automorphism group of $\mathbb{Z}[i]$.
2. Consider the ring $R = \mathbb{C}[X, Y]$.
   (a) Give an example of an ideal that is not prime.
   (b) Give an example of an ideal that is not principal.
   (c) Give an example of a prime ideal in R that is not maximal.

(d) Give an example of a maximal ideal.

(e) Is there a maximal ideal that is a principal ideal?

3. Describe the quotient ring $\mathbb{Z}[i]/(3)$. Is it a field? How many elements does it have?

4. Let $S \subset \mathbb{C}(X)$ be the set of rational functions $u(X) = \frac{f(X)}{g(X)}$ so that $g(0) \neq 0$.

   (a) Show that this is a ring but not a field.

   (b) Describe all ideals of S.

5. Consider the ring $R = \mathbb{Z}[\sqrt{3}]$.

   (a) Show that $\alpha \in R$ is a unit if and only if $N(\alpha) = \pm 1$ in $\mathbb{Z}$.

   (b) Show that $2 + \sqrt{3}$ is a unit in R.

   (c) Show that there are infinitely many units in R.

6. Find an integer quadratic diophantine equation that is solvable (rational integer solutions) if and only if the rational prime p factors in the ring $\mathbb{Z}[i\sqrt{5}]$.

7. Show that the polynomial $f(x) = x^5 + 6x^3 + 9x + 15$ is irreducible in $\mathbb{Z}[X]$.

8. Factor the polynomial $X^6 - 1$ into irreducible factors in $\mathbb{Z}[X]$.

9. Show that there is no rational function $u(X) = \frac{f(X)}{g(X)}$ in $\mathbb{C}(X)$ with $u^2(X) = (x - 3)$.