Honors Algebra II, Courant Institute, Spring 2022

http://www.math.nyu.edu/faculty/goodman/teaching/HonorsAlgebraII2022/HonorsAlgebraII.html Check the class forum for corrections and hints

Assignment 1

Due Thursday, Feb. 3.

Instructions

- Do not hand in a rough draft. Copy or type answers neatly and clearly. Points may be deducted for writing that is sloppy, has excessive cross-outs, or is hard to read.
- State facts precisely in clear language or notation. Put assertions in logical order. State clearly what the hypotheses and conclusions. Put the steps of an argument in logical order, including definitions. Points may be deducted for an incorrectly stated argument even if you seen to understand it. Clear mathematical exposition is an important goal for the class.
- Learn the Greek letters used in math. Learn their mathematical names and write them clearly.

Exercises

- From the textbook, Chapter 11: exercises 1.6, 1.8, 3.8. *Hint for 3.8.* Some writers write n.x for integer n and $x \in \mathbb{F}_p$ to represent the sum $x + \cdots + x$ (n summands). The point is that \mathbb{F}_p is an integral domain, so $y \cdot x \neq 0$ if $y \neq 0$ and $x \neq 0$. The notation n.x instead of $n \cdot x$ recognizes that $n \notin \mathbb{F}_p$. The binomial expansion of $(x + y)^p$ is a sum of terms of the form $\binom{p}{k}.x^{p-k}y^k$. The binomial coefficient is an integer. You can show that mp.x = 0 in \mathbb{F}_p for any integer m. This means that if p|n (meaning "p divides n"), then n.x = 0. Show that $p|\binom{p}{k}$ if k > 0 and k < p. For this, note that in the quotient excression for $\binom{p}{k}$, none of the integers in the denominator divides the p in the numerator.
- From the notes *Motivation*, exercises 3, 6, 7, 8, 9.