Intro. to Math. Modeling PROBLEM SET 6 Due February 28, 2005

1. Consider the linear system

$$\frac{dx}{dt} = 2x, \quad \frac{dy}{dt} = x - 3y.$$

(a) Write the system in the vector form $\frac{dv}{dt} = A \cdot v, v = \begin{pmatrix} x \\ y \end{pmatrix}$. What is the matrix A? What are its eigenvalues? Is the equilibrium $(0,0)$ stable or unstable?

(b) Write the general solution of the system in terms of the two eigenvalues and their corresponding eigenvectors.

(c) Write down the phase plane equation for the system. Find the general solution of the phase plane equation for $y(x)$. Hint: Let y(x) = $\frac{1}{5}x + s(x)$ and use separation of variables to solve for $s(x)$.)

(d) Compare the function $x(t), y(t)$ obtained in (b) with the solution of the phase plane equation you obtained in (c). Show that they are the same when $t$is eliminated.

2. Consider the model described at the bottom of page 252 for competition between two types of yeast. If $c/d > a/b$, sketch the phase plane and indicate the stable and unstable equilibria. What happens to this this system with time?

3. Consider the two-species competition model with $a = \sigma = 2, c = d = k = 1, b = 6$. Indentify the case. Verify that there is an equilibrium (x_+, y_+) with both species populations non-zero. Sketch the phase plane. Test the stability of the non-zero equilibrium (x_+, y_+) by finding the eigenvalues of the local A matrix. Verify that your conclusion matches your phase plane sketch.