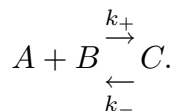


1. Consider a chemical reaction where A and B combine to produce C with a rate constant k_+ , and simultaneously C can decompose into A and B with a rate constant k_- . In symbols



(a) Three ODEs describe the kinetics of this reaction. One of them is $\frac{dA}{dt} = -k_+AB + k_-C$. What are the other two equations?

(b) What can you say about the evolution of $A - B$, $A + C$, $B + C$?

(c) Give a single ODE for the evolution of C in the form $\frac{dC}{dt} = F(C)$. Assume that $C(0) = 0$.

(d) Assume $A(0) = 2$, $B(0) = 1$ and $k_+ = 2$, $k_- = 1$, and that $C(0) = 0$. What is the value of C when the reaction terminates? Determine this by sketching the $F(C)$ and observing where the stable equilibrium is.

(e) Does the unstable root of $F(c) = 0$ make any sense chemically? Explain.

2. This is an example of analysis of a Turing instability. Isolated cell states are determined by the concentrations of two chemicals, $(x(t), y(t))$. They satisfy the following chemical kinetic equations:

$$\frac{dx}{dt} = F(x, y) = -3x + 10y - 4y^2; \quad \frac{dy}{dt} = G(x, y) = -x + 8y - 3xy.$$

(a) Show that isolated cells can have three equilibrium states. (One of these is $(x_e, y_e) = (56/27, 7/6)$.)

(b) Compute the matrix

$$A = \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{pmatrix}$$

and use it to determine the linear stability or instability of the equilibrium states of the cells. (A cell is stable if $T < 0$ and $D > 0$ where T, D are the trace and determinant of A evaluated at the equilibrium. You will find that only one cell equilibrium is stable.)

(c) The cells are now formed in a ring, and communicate by diffusion, with diffusion coefficients μ_x, μ_y , as in the notes. What is the function $\bar{D}(k)$ for this model (in the notation of the notes), if the resting state is the unique stable equilibrium?

(d) If $\mu_y = 1$, how large must μ_x be for a diffusive instability to occur?

(e) If μ_x has the value of the threshold for instability given in (d) (and $\mu_y = 1$), what is the threshold k , i.e. the number k_c of the notes?

(f) If $\mu_x = 6$ and $\mu_y = 1$, what is the window of instability $k_{min} < k < k_{max}$?

(g) Again let $\mu_x = 6$ and $\mu_y = 1$. Suppose that the cells form a ring of circumference $L = 1$. Then the cells begin to divide so that L increases from 1. At what value of L will a diffusive pattern form (i.e. at what value of L is the ring diffusively unstable)?