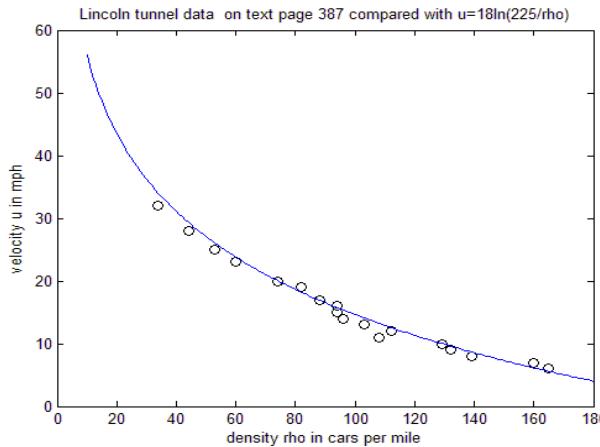


1. Lincoln tunnel data given on page 387 of text for velocity versus density is plotted in the figure below and compared with the relation  $u = a \ln(\rho_{max} / \rho)$  with  $a = 18$  mph and  $\rho_{max} = 225$  cars per mile. The good fit suggests that we try this relation for  $u(\rho)$  in our problem of a red light turning green. Keep  $a, \rho_{max}$  as arbitrary in the calculations to follow. Note that this relation gives  $u = \infty$  when  $\rho = 0$ . We will accept this, since the cars will not reach small density. This *will* mean that  $v(0) = dq/d\rho(0) = \infty$  so that the positive  $\rho$  axis becomes the right-hand boundary of the expansion fan.



Assume a red light is on at  $x = 0$ . At time  $t = 0$  the red light turns green.

- Write down the function  $q(\rho)$  for this model.
- Determine  $v(\rho) = dq/d\rho$  for this model.
- What is  $v$  at  $\rho = \rho_{max}$  in mph?
- Write down the traffic flow equation for this model. Solve for  $\rho(x, t)$  in the expansion fan by setting  $\rho(x, t) = R(\eta)$ ,  $\eta = x/t$ .
- Given  $u(\rho)$  in this model and the result found in (d), calculate the velocity of a car in the expansion fan as a function of  $x, t$ . Setting this velocity equal to  $dx/dt$ ,  $x(t)$  being car position, solve the resulting differential equation (using an integrating factor), to find the path of the car which at  $t = 0$  was located a distance  $D$  behind the red light.
- Determine the distance behind the red light of cars that will make it through a green light which lasts for  $t_G$  hours. This should be a function of  $t_G$  and  $a$  alone. With  $t_G = 1/30$  hours and  $a = 18$  mph, what is the length in miles?

- A  $t = 0$  the traffic on a one-lane road has a uniform density of  $\rho$  cars/mile in  $x < 0$ , has a density of  $\rho = 50 + 100x$  cars/mile in on  $0 < x < 1$  mile, and has density  $\rho = 150$  for  $x > 1$  where  $x$  is in miles. Assume that  $u_{max} = 40$  mph and  $\rho_{max} = 250$  cars/mile, so that  $u = 40(1 - \rho/250)$ .

(a) Determine  $q(\rho)$  and  $v(\rho) = dq/d\rho$ . Write down the equation for the characteristics in each region ( $x_0 < 0, 0 < x_0 < 1, x_0 > 1$ ).

(b) Sketch the characteristics in the  $x - t$  plane for the three regions. Verify that the characteristics coming out of the segment  $0 < x < 1$  intersect at a point. This marks the initiation of a shock. At what time (in minutes) and where ( $x = ?$  miles) does the shock first form?

(c) Determine the velocity of the shock, and the jump in density across the shock moving from left to right, and write down the equation for the path of the shock ( $x_{shock}$  as a function of time). Sketch the shock in the  $x - t$  plane.

(d) Describe the path of the car which at  $t = 0$  is located at  $x = -.5$  miles, indicating its velocity as a function of time. Where is the car when the shock first forms? When and where does it encounter the shock? How fast does it go after it passes the shock? What is the equation of the car's position after it passes the shock?

3. How would you model the closure of one lane of a one-way, two-lane road? (Suggestions: A two-lane road can be thought of as a road with the same  $u_{max}$  as a one-lane road but with twice the maximum density. The problem is similar to that of encountering a change of pavement but the two flow-density relations are different. To take a specific case with  $q = u_{max}\rho(1 - \rho/(2\rho_{max}))$  on the 2-lane road, take  $u_{max} = 60$  mph,  $\rho_{max} = 300$  cars/mile, and let the oncoming density on the 2-lane road be 6000 cars/hour.)