Derivation of scaling of the lower deck

\[ u_x + v_y = 0 \]  
Gives  
\[ U/X = V/Y, \quad (1) \]

\[ uu_x \] balancing \( \frac{1}{R} u_{yy} \) gives  
\[ RU = X/Y^2, \quad (2) \]

In these the capital letters (except for \( R \)) represent scaling factor order relative to \( R \).  
The main deck supplies the pressure of order \( V/U \), so we have from balancing \( p_x \) and \( uu_x \)

\[ V = U^3, \quad (3) \]

Finally, the upstream boundary condition is that \( u \) match with a boundary layer, for which \( \partial u / \partial y \sim R^{1/2} \). Thus

\[ U^2 = Y^2 R, \quad (4) \]

Now from (1) and (3),

\[ U^2 = Y/X, \quad (5) \]

From (4) and (5)

\[ XYR = 1, \quad (6) \]

From (5) and (2),

\[ R^2 Y^5 = X^3 \quad (7) \]

Solving (6) and (7) for \( X, Y \) as function of \( R \), we obtain

\[ X = R^{-3/8}, \quad y = R^{-5/8} \]

We then see that

\[ U = R^{-1/8}, \quad V = R^{-3/8}, \quad P = R^{-1/4} \]

in the lower deck.