Fluid Dynamics I PROBLEM SET 6 Due November 14, 2005

1. Using the method of Blasius for obtaining moment, as outlined in class, show that the moment of an arbitrary body in 2D potential flow is given by

$$M = -\frac{1}{2}\rho\Re[\int_C z(dw/dz)^2 dz]$$

where $\Re$ denotes the real part and C is any simple contour about the body. Using this, verify that the circular cylinder flow with vortex of strength Γ at its center experiences zero moment. (Use the residue method.).

2. Consider the Joukowski airfoil with $\zeta_0 = bi \; a > b > 0$. (a) Show that the airfoil is an arc of the circle with center at $(0, -(a^2 - b^2)i/b$ and radius $(a^2 + b^2)/b$. (b) With Kutta condition applied to the trailing edge, at what angle of attack (zas a function of b)is the lift zero?

3. Let the airfoil parameters other than chord (i.e. k, β) be independent of y, the coordinate along the span of the wing. Also, assume the planform is symmetric about the line $x = 0$ in the $x - y$ plane. Using Prandtl's lifting-line theory, show that for a given lift the minimal induced drag occurs for a wing having an elliptical planform. Show in this case that the coefficient of induced drag $C_{D_i} = 2 \times drag/(\rho U^2 S)$ and lift coefficient $C_L = 2 \times lift/(\rho U^2 S)$ are related by

$$C_{D_1} = C_L^2/(\pi A).$$

Here S is the wing area and A is the aspect ratio $4b^2/S$. (Some of the WW II fighters, notably the Spitfire, adopted an approximately elliptical wing.)

4. A 3D body D move steadily with velocity $\mathbf{U}$. The flow is a potential flow exterior of the body and $\frac{\partial\phi}{\partial n} = 0$ on the body surface ∂D. Given that for large $R^2 = x^2 + y^2 + z^2$ the perturbation potential ϕ ($\mathbf{u} = \nabla\phi - \mathbf{U}$ relativeto the moving body) decays like

$$\phi = -\frac{a}{R} - \frac{\mathbf{A}\cdot\mathbf{R}}{R^3} + O(R^{-3}),$$

where $a, \mathbf{A}$ are constants (scalar and vector respectively), show that necessarily $a = 0$. (Note: $\int_{\partial D} \mathbf{n}\cdot\mathbf{U} dS = 0$.)