Fluid Dynamics I PROBLEM SET 5 Due October 31, 2005

1. Complete the proof of zero drag of the Rankine fairing using the momentum integral method, as outlined in class.

2. In the Butler sphere theorem, we needed the following result: Show that $\Psi_1(R,\theta) \equiv \frac{R}{a}\Psi(\frac{a^2}{R},\theta)$ is the streamfunction of an irrotational, axisymmetric flow in spherical polar cordinates, provided that $\Psi(R,\theta)$ is such a flow. (Hint; Show that $L_R\Psi_1(R,\theta) = L_{R_1}\Psi(R_1,\theta)$, where $R_1 = a^2/R$. Here $L_R\Psi = R^2\frac{\partial^2\Psi}{\partial R^2} + \sin\theta\frac{\partial}{\partial\theta}\big(\frac{1}{\sin\theta}\frac{\partial\Psi}{\partial\theta}\big)$.)

3. Show that in spherical polar coordinates, the streamfunction Ψ for a source of strength Q, placed at the origin, normalized so that $\Psi = 0$ on $\theta = 0$, is given by $\Psi = \frac{Q}{4\pi}(1-\cos\theta)$. (Recall $u_R = \frac{1}{R^2\sin\theta}\frac{\partial\Psi}{\partial\theta}, u_\theta = \frac{-1}{R\sin\theta}\frac{\partial\Psi}{\partial R}$.) Find the streamfunction in spherical polars for the airship model consisting of equal source and sink of strength Q, the source at the origin and the sink at $R = 1, \theta = 0$, in a uniform stream with streamfunction $\frac{1}{2}UR^2\sin\theta^2$. The sink will thus involve the angle with respect to $R = 1, \theta = 0$. Use the law of cosines ($c^2 = a^2 + b^2 - 2ab\cos\theta$ for a triangle with θ opposite side c) to express Ψ in terms of R, θ.

4. (a) Show that the complex potential $w = Ue^{i\alpha}z$ determines a uniform flow making an angle α with respect to the x-axis and having speed U.

(b) Describe the flow field whose complex potential is given by

$$w = Uze^{i\alpha} + \frac{Ua^2e^{-i\alpha}}{z}.$$

5. Recall the following rule for the motion of point vortices in two dimensions: Each vortex moves with the velocity equal to the sum over the velocities contributed by all other vortices, at the point in question. That is, now using the complex potential.

$$dz_k(t)/dt = \overline{w'(z_k)}, w_k = \sum_{j=1, j\neq k}^{N} \gamma_j \log(z - z_j(t)), \gamma = -i\Gamma/2\pi,$$

where the strengths are γ_i and the positions are $z_i(t)$. (a) Using this rule, show that two vortices of equal strengths rotate on a circle with center at the midpoint of the line joining them, and find the speed of their motion.

(b) Show that two vortices of strengths γ and $-\gamma$ move together on straight parallel lines perpendicular to the line joining them. Again find the speed of their motion.