1. For potential flow over a circular cylinder as discussed in class, with pressure equal to the constant p_∞ at infinity , find the static pressure on surface of the cylinder as a function of angle from the front stagnation point. (Use Bernoulli's theorem.) Evaluate the drag force (the force in the direction of the flow at infinite which acts on the cylinder), by integrating the pressure around the boundary. Verify that the drag force vanishes. This is an instance of *D'Alembert's paradox*, the vanishing of drag of bodies in steady potential flow.

2. For an ideal inviscid fluid of constant density, no gravity, the conservation of mechanical energy is studied by evaluating the time derivative of total kinetic energy in the form

$$\frac{d}{dt} \int_D \frac{1}{2} \rho |\vec{u}|^2 dV = \int_{\partial D} \vec{F} \cdot \vec{n} dS.$$

Here D is an arbitrary fixed domain with smooth boundary ∂D. What is the vector $\vec{F}$? Interpret the terms of $\vec{F}$ physically.

3. An open rectangular vessel of water is allowed to slide freely down a smooth frictionless plane inclined at an angle α to the horizontal, in a uniform vertical gravitational field of strength g. Find the inclination to the horizontal of the free surface of the water, given that it is a surface of constant pressure. We assume the fluid is at rest relative to an observer riding on the vesel. (Consider the acceleration of the fluid particles in the water and balance this against the gradient of pressure.)

4. Water (constant density) is to be pumped up a hill (gravity $= (0, 0, -g)$) through a pipe which tapers from an area A_1 at the low point to the smaller area A_2 at a point a vertical distance L higher. What is the pressure p_1 at the bottom, needed to pump at a volume rate Q if the pressure at the top is the atmospheric value p_0? (Express in terms of the given quantities. Assuming inviscid steady flow, use Bernoulli's theorem with gravity and conservation of mass. Assume that the flow velocity is uniform across the tube in computing fluid flux and pressure.)

5. For a *barotropic fluid*, pressure is a function of density alone, $p = p(\rho)$. In this case derive the appropriate form of Bernoulli's theorem for steady flow without gravity. If $p = k\rho^\gamma$ where γ, k are positive constants, show that $q^2 + \frac{2\gamma}{\gamma-1} \frac{p}{\rho}$ is constant on a streamline, where $q = |\vec{u}|$ is the speed.

6. Water fills a truncated cone as shown in the figure. Gravity acts down (the direction $-z$). The pressure at the top surface, of area A_2 is zero. The height of the container is H. At $t = 0$ the bottom, of area $A_1 < A_2$, is abruptly removed and the water begins to fall out. Note that at time $t = 0+$ the pressure at the bottom surface is also zero. The water has not moved but the acceleration is non-zero. We may assume the resulting motion is a potential flow. Thus the potential $\phi(z, r, t)$ in cylidrical polars has the Taylor series $\phi(r, z, t) = t\Phi(r, z) + O(t^2)$, so $d\phi/dt = \Phi(r, z) + O(t)$. Using these facts, set up a mathematical problem for determining the pressure on the inside surface of cone at $t = 0+$. You should specify all boundary conditions. You do not have to solve the resulting problem, but can you guess what the surfaces $\Phi =$ constant would look like qualitatively? What is the force felt at $t = 0+$ by someone holding the cone, in the limits $A_1 \to 0$ and $A_1 \to A_2$?