Fluid dynamics of animal locomotion PROBLEM SET 5 Due November 24, 2004

1. In the following two examples the object is to compute the average x-directed thrust given that the side forces on the fish are somehow in equilibrium.vWe will compare the thrust result for small amplitudes with that for finite amplitudes. The relevant formulas are

$$\langle T\rangle = m(L)\langle wW - \frac{1}{2}w^2\rangle_{x=L},\ W = h_t, w = h_t + Uh_x,$$

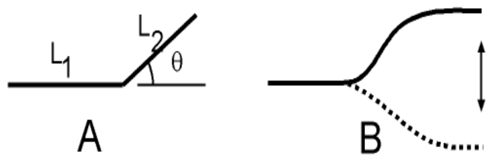
for small amplitudes and

$$\langle T\rangle = m(L)\langle wz_t - \frac{1}{2}w^2 x_s\rangle_{s=L}$$

for finite amplitudes.

Problem A: The moveable part of the body is a straight riigid segment of length L_2 attached to a segment of length L_1. The angle θ is given by $\theta = \theta_0 \sin\omega t$.

Problem B: The caudal fin moves in such a way that the tangent vector $\vec{t}$ at the rear edge of the caudal fin (i.e. at $s = L$) is always parallel to the direction of swimming, and the position of the edge is given by $z = H_0 \sin\omega t$.



2. Discuss the efficiencies for both small and finite amplitudes in the two examples of problem 1.

3. This problem suggests how Lighthill's thrust for a slender fish in the small amplitude theory can be derived from momentum arguments alone. Consider the momentum M contained within the fluid external to thd body and upstream of the plane Σ passing through the edge $x = L$ of the caudal fin. The swimming is in the -x and the observed is fixed with the fluid at infinity (stationary). For a slender fish the x-momentum flux at the fish is negligible so we have

$$\frac{dM}{dt} = -T = -\int_{fish} p\vec{n}dS + \vec{i}\int_{\Sigma} p\ dS,$$

where T is the thrust developed by the fish. The integral over Σ may be evaluated using the unsteady Bernoulli theorem, the phi_t being negligible because ϕ is odd in z. Also the $\frac{1}{2}w^2$ may be evaluated in terms of the apparent mass and w at $x = L$. The integral of the fish can be evaluated in terms of vortex and inertial forces. The total y-vorticity in a section of depth $2c$is

$$\int_{-c}^{+c} \gamma_y dy = -\frac{\partial(wm)}{\partial x}$$

and so the corresponding vortex force in the x-ditrection exerted on the fish is $w\frac{\partial(wm)}{\partial x}$. Similarly the inertial force has an x-component $-h_x D(mw)$. Thus

$$T = \int_0^L [w\frac{\partial(wm)}{\partial x} - h_x D(mw)]dx - [\frac{1}{2}mw^2]_{x=L}.$$

Show that this gives Lighthill's result for small amplitudes.