1. Define an efficiency of the ciliated sphere by

$$\eta = \frac{6\pi\mu r_0 U_{sc}^2}{W}.$$

Where W is computed by integrating $W_s = \mu\omega^2 k(a^2 + b^2)$ over the surface of the sphere. Here $U_{sc} = -\frac{1}{3}\omega k(b_0^2 + \frac{10}{3}a_0 b_0 \cos\phi - a_0^2)$ is the swimming velocity of the sphere. Show that, if a^2 and b^2 is distributed over the sphere in proportion to $\sin\theta$, $(a^2, b^2) = \sin\theta(a_0^2, b_0^2)$, optimal efficiency is about (to closest integer)

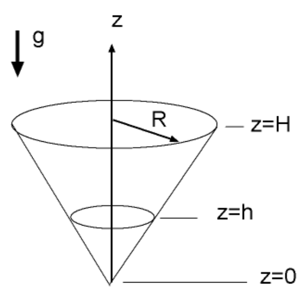
$$5\Big(\frac{a_0^2 + b_0^2}{r_0\lambda}\Big), \lambda = \frac{2\pi}{k}.$$

For a ciliate such as *Opalina*, we can take $r_0 = 80\ \mu$, $\lambda = 40\ \mu$m, and $a_0^2 + b_0^2$ =25 μm^2, in which case $\eta_{max} \approx 4$ percent. (Hint: see the discussion of efficiency of the sheet in the notes on ciliary propulsion.)

2. An Euler flow with no vortex force is called a *Beltrami field*. Show that $\vec{u} = \nabla \times \nabla \times \vec{A} + \alpha\nabla \times \vec{A}$ is a Beltrami field for any $\vec{A}$ satisfying $\nabla^2\vec{A} + \alpha^2\vec{A} = 0$.

3. Water fills the right circular cone shown in the figure, gravity acting down. At time $t = 0$, the bottom section at $z = h$ is sliced off, so that water flows out the bottom. At time $t = 0+$ however, the water has not moved, but the pressure at *both* the bottom section $z = h$ and the top section $z = H$ is the ambient pressure p_0. The question is, how would you determined the pressure distribution on the inner surface of the cone, at time $t = 0+$? Notice that if $h = 0$ the pressure distribution is that of the static system, $p = p_0 + \rho g(H - z)$, where $\rho =$ the density of water. Also, if h is close to H, the pressure is just p_0 throughout.

Make use of the unsteady Bernoulli theorem, assuming the developing flow is potential. Expand the potential as a Taylor series in time.You should formulate a problem to be solved for ϕ, but you do not have to solve this problem or to find the pressure distribution explicitly.



4. The potential flow of a uniform stream $\vec{U} = (U(t), 0, 0)$ around a fixed sphere $r = a$ has the potential

$$\phi = U(t)\big(x + \frac{a^3 x}{2r^3}\big).$$

What is the potential of a sphere moving through fluid otherwise at rest with speed $(U(t), 0, 0)$? Evaluate the pressure using the unsteady Bernoulli theorem, compute the pressure force on the sphere, and compare your result against our general expression for apparent mass.

5. Show that the apparent mass tensor $\vec{M}$ is symmetric, $M_{ij} = M_{ji}$. Do this by using the expression $M_{ij} = -\rho\int \Phi_i n_j dS$ tken over the surface of the body. Then consider $\int(\Phi_j n_i - \Phi_i n_j)dS$, using the boundary condition on S to write the integral in the form

$$\int \big(\Phi_j \frac{\partial\Phi_i}{\partial x_k} - \Phi_i \frac{\partial\Phi_j}{\partial x_k}\big) n_k dS.$$

Then apply green's theorem to the external domain with $\Phi = O(r^{(1-N)})$ at infinity.

6. Show that a homogeneous, axially symmetric, neutrally buoyant squirmer, starting from rest with a time-reversible boundary motion, cannot swim along its symmetry axis in an inviscid fluid. (See the discussion of squirming along a line in Chapter 3.)